Coherent quantum phase slips in disordered Josephson junction chains

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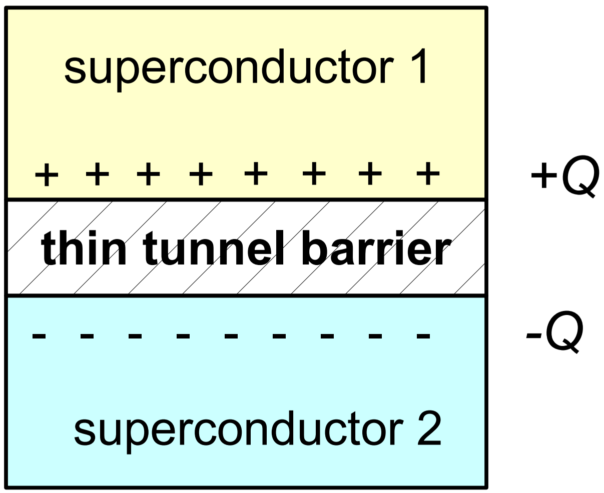
Josephson junction



Superconducting phase difference $\theta = \phi_1 - \phi_2$

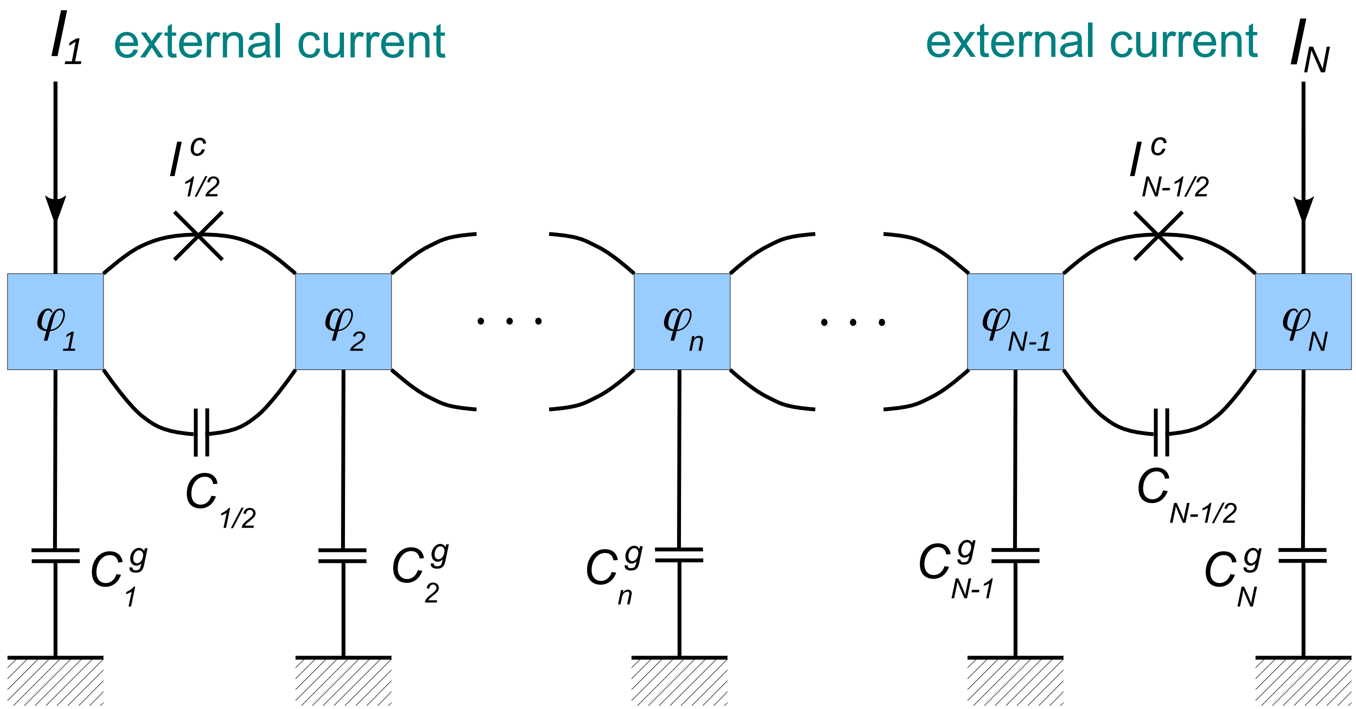
$$\hat{H} = \frac{\hat{Q}^2}{2C} + E_J(1 - \cos\hat{\theta})$$

$$[\hat{Q}, \hat{\theta}] = 2ie$$

$$E_J \gg E_C \equiv \frac{(2e)^2}{C} \quad \Rightarrow \quad 1 - \cos\hat{\theta} \approx \frac{\hat{\theta}^2}{2}$$

harmonic oscillator

Josephson junction chain



a small capacitance between the islands and the ground

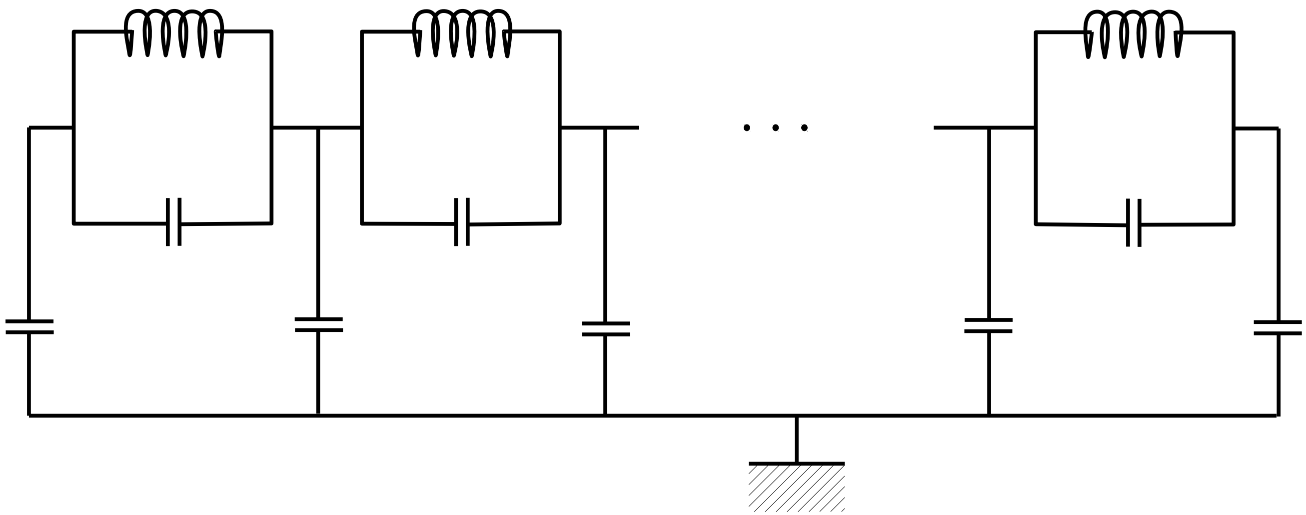
Typically,

$$C \gg C^g$$

Small oscillations of the phase

$$\sin(\phi_n - \phi_{n+1}) \to \phi_n - \phi_{n+1}$$

Josephson current ~ inductance



$$Y_{n-1/2}(V_n - V_{n-1}) + Y_{n+1/2}(V_n - V_{n+1}) - i\omega C_n^g V_n = 0$$ linear "wave" equation

$$Y_{n+1/2}(\omega) = -\frac{1}{i\omega L_{n+1/2}} - i\omega C_{n+1/2} \cdot$$ complex admittance of the junction

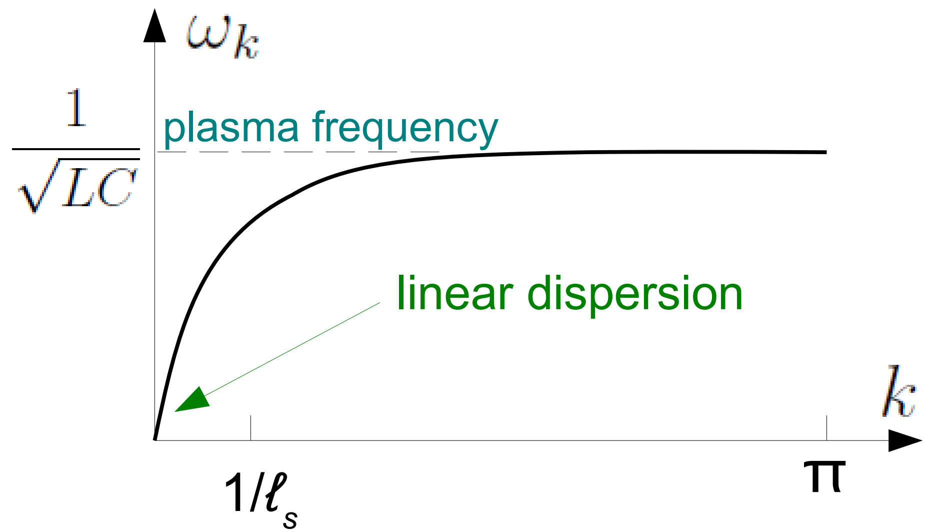
$$\frac{d\phi_n}{dt} = -2eV_n$$

Plasma waves in Josephson chains

Infinitely long chain: $V_n \propto e^{ikn}$

$$\omega_k = \frac{1}{\sqrt{LC}}\sqrt{\frac{4\sin^2(k/2)}{4\sin^2(k/2)+C^g/C}}$$

Mooij-Schön modes



screening length $\ell_s \equiv \sqrt{C/C^g} \gg 1$

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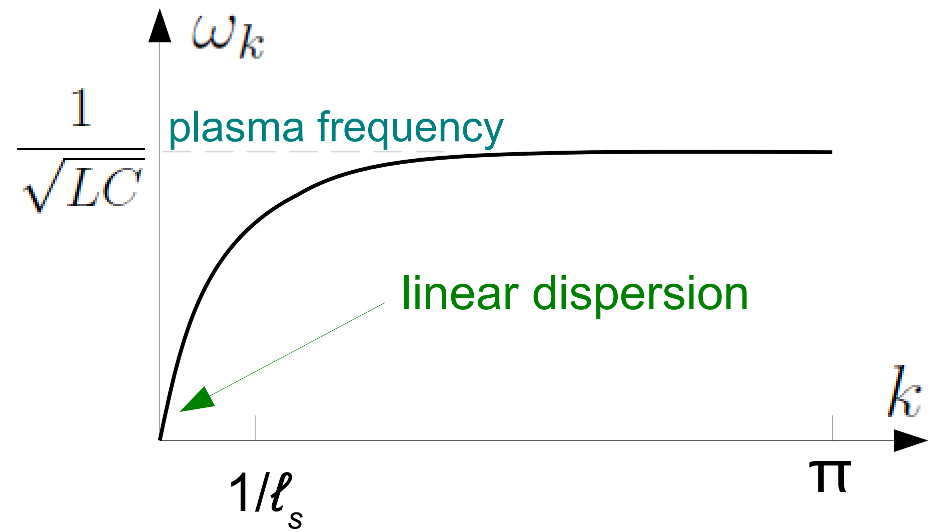
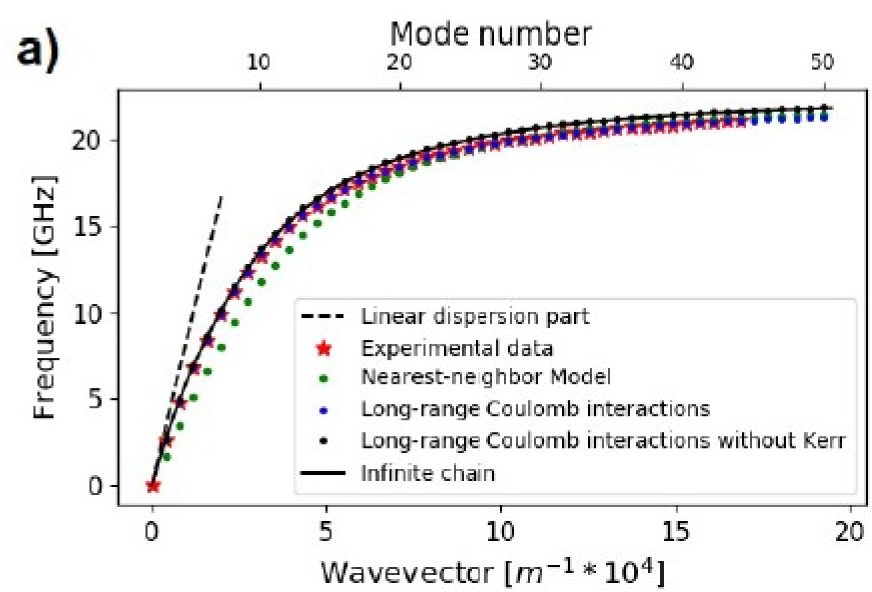


screening length $\ell_s \equiv \sqrt{C/C^g} \gg 1$

Finite length, N junctions:

$$k = 0, \frac{\pi}{N}, \frac{2\pi}{N}, \ldots, \frac{(N-1)\pi}{N}$$

Measured resonances
in the microwave transmission coefficient
of a 200-junction chain
Yu. Krupko et al., PRB **98**, 094516 (2018)



Random offset charges

$$\hat{H} = \frac{1}{2}\sum_{n,n'} C^{-1}_{nn'}\left(\hat{q}_n - 2e\kappa_n\right)\left(\hat{q}_{n'} - 2e\kappa_{n'}\right) - \sum_{n=0}^{N-1} E_{Jn}\cos(\phi_{n+1} - \phi_n)$$

Classically: **no effect** on normal modes

Quantum-mechanically: energy levels sensititve to the fractional part of $\boldsymbol{\kappa_n}$

Random spatial modulation of areas

Josephson energy, junction capacitance ∝ **junction area**

$E_{Jn} = E_J(1+\zeta_n), \quad C_n = C(1+\zeta_n)$ weak relative modulation of the junction areas $\left\langle \zeta_n^2 \right\rangle = \sigma_J^2 \ll 1$

$C_n^g = C^g(1+\eta_n),$ weak relative modulation of the ground capacitances $\left\langle \eta_n^2 \right\rangle = \sigma_g^2 \ll 1$

All normal modes are localized

Long chains: localization length from the DMPK equation

$$\xi = \frac{2}{\sigma_J^2 + \sigma_g^2}\,\frac{4}{k^2}$$

diverges at $k \to 0$ (standard for Goldstone modes)

Short chains $N \ll \xi$: random perturbative shifts of the discrete frequencies

$$\left\langle \delta\omega_k^2 \right\rangle = \frac{3/8}{LC}\,\frac{\sigma_J^2 + \sigma_g^2}{N}\,\frac{k^2\ell_s^2}{(1+k^2\ell_s^2)^3}$$

Basko & Hekking, PRB **88**, 094507 (2013)

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Quantum fluctuations in the equilibrium state of a thin superconducting loop

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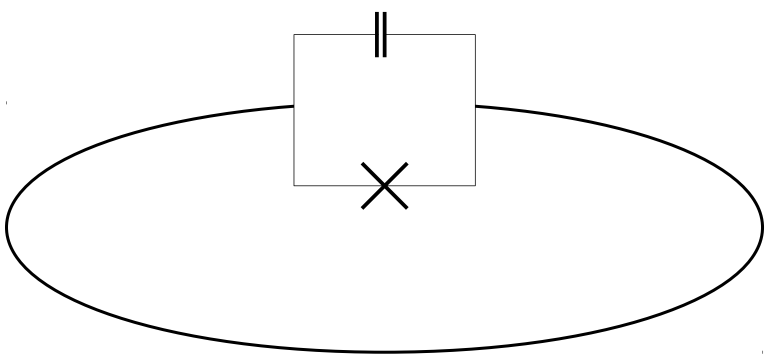


Mooij-Schön modes of the loop

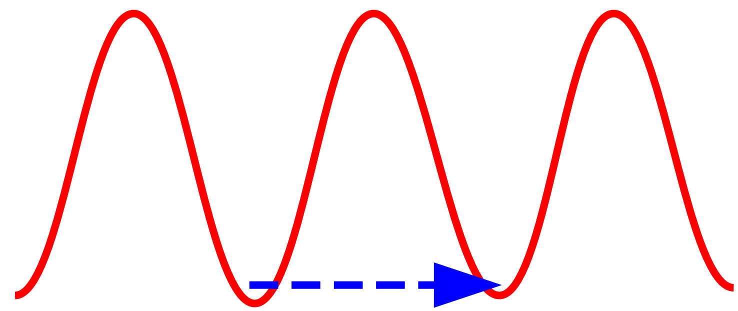
physical Ohmic environment for QPS

QPS amplitude:

$$A \sim \frac{v_{\rm pl}}{\ell^*} e^{-g \ln(L/\ell^*)}$$

$$\ell_* = g v_{\rm pl}/(\pi E_J)$$

JJ closed in a superconducting loop:
quantum tunneling in the $\cos\theta$ potential



no energy to dissipate

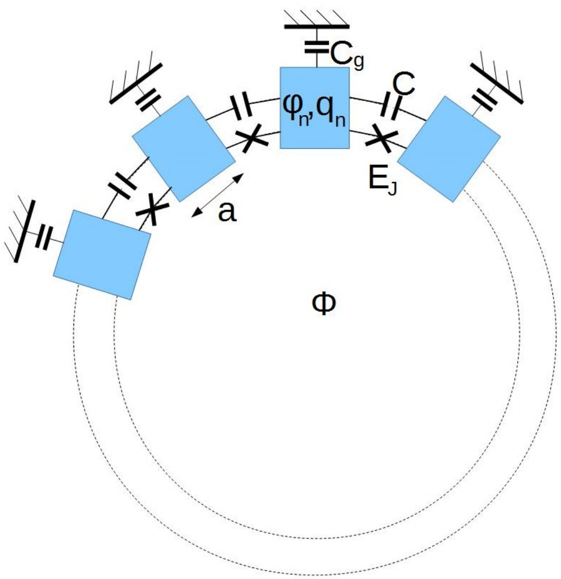
coherent QPS

$v_{\rm pl}$ – Mooij-Schön mode velocity
g – dimensionless loop admittance
in units of $2e/(\pi\hbar)$

Coherent quantum phase slips

Matveev, Larkin, Glazman, PRL **89**, 096802 (2002)
Rastelli, Pop, Hekking, PRB **87**, 174513 (2013)

JJ ring pierced by magnetic flux:



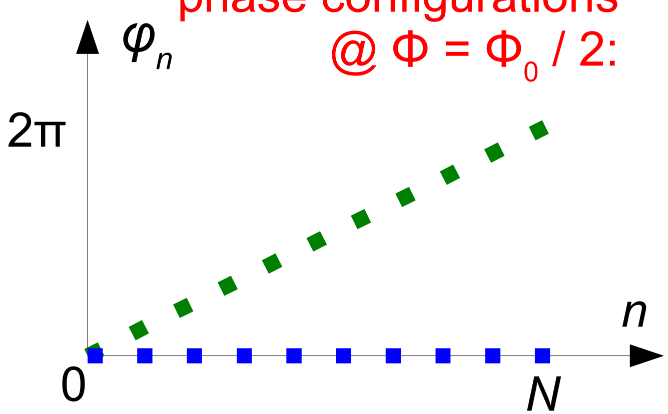
Tunneling amplitude from an instanton calculation

Imaginary-time Lagrangian:

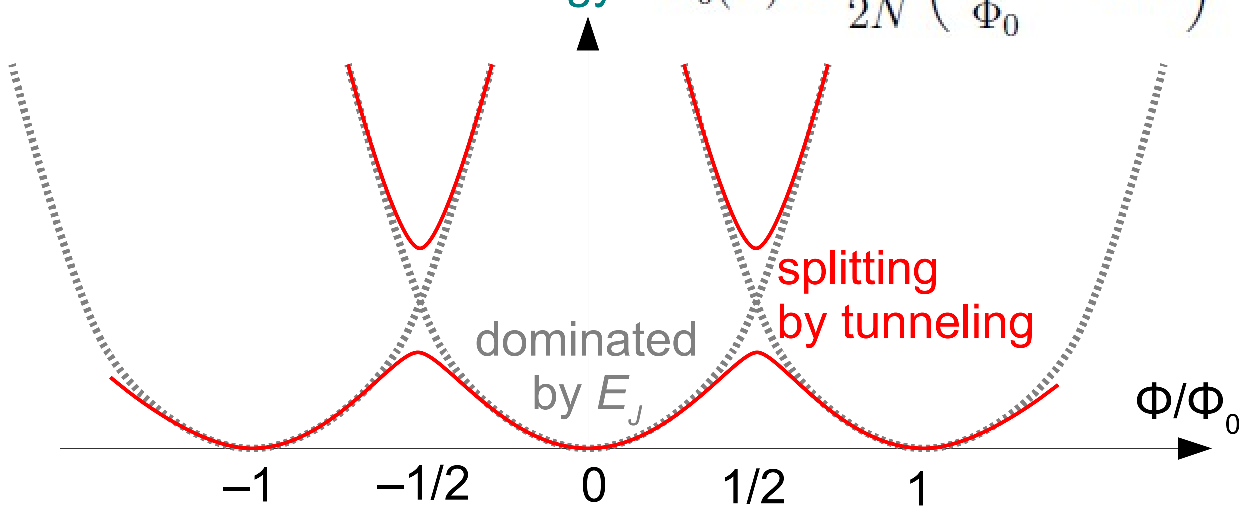
$$\mathcal{L} = \sum_n \left[\frac{C_g}{8e^2}\,\dot{\varphi}_n^2 + \frac{C}{8e^2}\left(\dot{\varphi}_{n+1} - \dot{\varphi}_n\right)^2 \right]$$ kinetic energy

$$- \sum_n E_J \cos\left(\varphi_{n+1} - \varphi_n + \frac{2\pi\Phi}{N\Phi_0} \right)$$ potential energy

Tunneling between two degenerate classical phase configurations @ $\Phi = \Phi_0 / 2$:



Ground state energy $E_0(\Phi) \approx \frac{E_J}{2N}\left(\frac{2\pi\Phi}{\Phi_0} - 2\pi m\right)^2$



QPS in a spatially uniform chain

1. Phase winding on one of the junctions
2. Phase readjustment on the length ~ ℓ_s
3. Phase readjustment in the rest of the chain

} sensitive to the phase normal modes

QPS amplitude:

amplitudes on different junctions added coherently (assuming no offset charges)

$$A = \frac{4N}{\sqrt{\pi}}\left(E_J^3 E_C\right)^{1/4} \exp\left\{-8\sqrt{\frac{E_J}{E_C}} - g\left[\ln\frac{N}{\ell_s} - 2.43 + O(1/\ell_s)\right]\right\}$$

Matveev, Larkin, Glazman, PRL **89**, 096802 (2002)

Hekking & Glazman, PRB **55**, 6551 (1997); Rastelli, Pop, Hekking, PRB 87, 174513 (2013)

Svetogorov *et al.*, PRB **97**,104514 (2018)

$$E_C = \frac{(2e)^2}{C}$$ junction charging energy << E_J

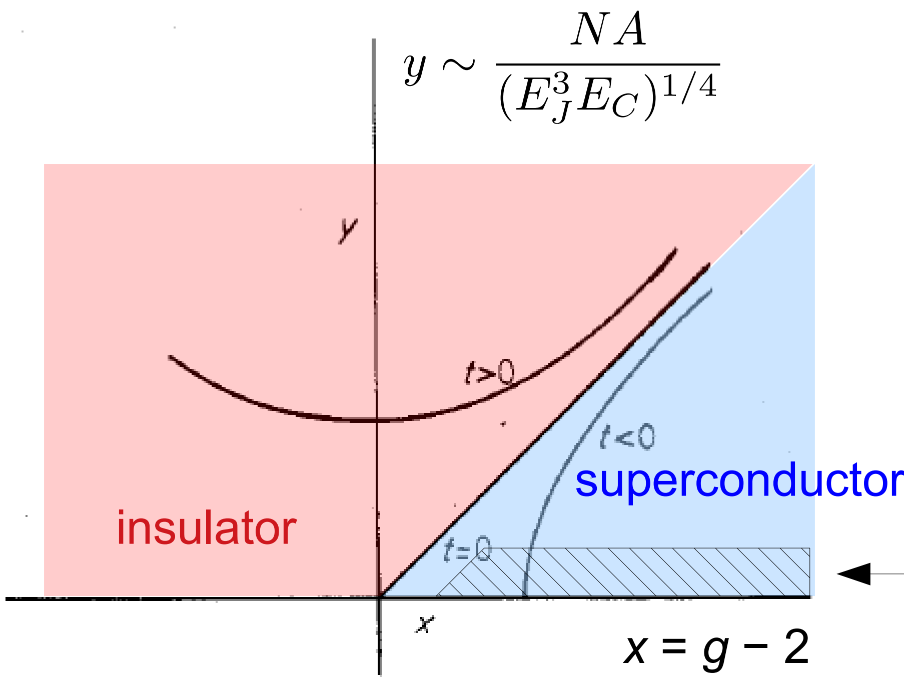
$$\ell_s \equiv \sqrt{C/C^g} \gg 1$$

$$g \equiv \pi\sqrt{\frac{E_J}{(2e)^2/C_g}} > 2$$ otherwise insulator

Bradley & Doniach, PRB **30**, 1138 (1984)
Korshunov, JETP **68**, 610 (1989)

dimensionless admittance of the chain

Kosterlitz-Thouless RG



J. M. Kosterlitz, J. Phys. C **7**, 1046 (1974)

$$\frac{dx}{d\ln N} = -\frac{(x+2)^2 y^2}{4}$$
$$\frac{dy}{d\ln N} = -xy$$

Our region of interest:

y << 1

RG flow gives $g \ln \frac{N}{\ell_s}$

Effect of a periodic spatial modulation

Josephson energy, junction capacitance ∝ **junction area**

$$E_{Jn} = E_J(1+\zeta_n), \quad C_n = C(1+\zeta_n)$$

weak relative modulation of the junction areas

$$\zeta_n = t_\zeta \cos\frac{2\pi n}{a}$$

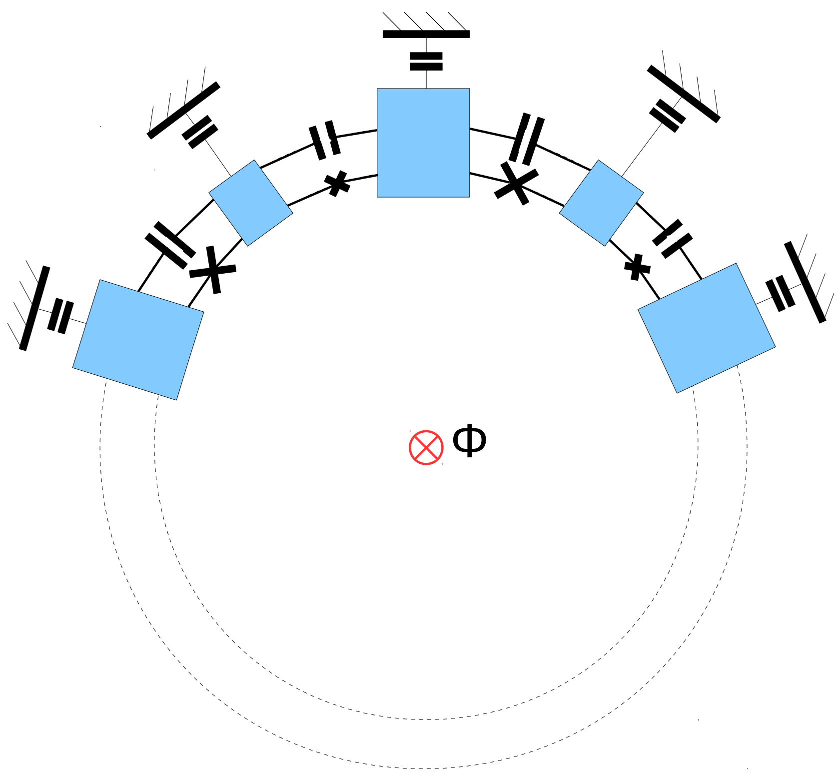
$$C^g_n = C^g(1+\eta_n),$$

weak relative modulation of the ground capacitances

$$\eta_n = t_\eta \cos\frac{2\pi n}{a}$$

modulation depth << 1

modulation period >> 1



Mooij-Schön modes are modified

structured environment

Effect on the QPS amplitude?

Effect of a periodic spatial modulation

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modulation depth << 1

modulation period >> 1

wave equation with modulation

↓

correction to mode wave functions

↓

correction to the QPS amplitude on junction *n*:

local admittance at the QPS position

$$A_n = \frac{4}{\sqrt{\pi}}\left(E_J^3 E_C\right)^{1/4} \exp\left\{-8\sqrt{\frac{E_{J,n}}{E_{C,n}}} - \bar{g}\left(\ln\frac{N}{\ell_s} - 2.43\right) - \delta g_n \ln\frac{0.10\,a}{\ell_s}\right\}$$

purely local

determined by modes $1/N < k < 1/\ell_s$

If $a \gg \ell_s$ only; otherwise $\sim a^2/\ell_s^2$

determined by modes $1/a < k < 1/\ell_s$

Svetogorov *et al.*, PRB **97**,104514 (2018)

Effect of a random spatial modulation

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QPS amplitude on junction *n*: $A_n \propto e^{-S_n - 2\pi i(\kappa_1 + \ldots + \kappa_n)}$ random offset charges

Ivanov *et al*., PRB **65**, 024509 (2001)
Matveev et al., PRL **89**, 096802 (2002)

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Homogeneous chain: $$S = 8\sqrt{E_J/E_C} + g\left(\ln\frac{N}{\ell_s} - 2.43\right)$$

determined by the slipping junction

determined by modes with $1/N < k < 1/\ell_s$

Disordered chain: $$\langle \delta S_n^2 \rangle = \left\langle \delta\left(8\sqrt{E_J/E_C}\right)^2 \right\rangle + \frac{\sim 1}{\ell_s}\langle (\delta g)^2 \rangle$$

determined by modes with $k \sim 1/\ell_s$

Coherent QPS amplitude is **NOT** sensitive to Anderson localization of the normal modes

Svetogorov & Basko, PRB **98**, 054513 (2018)

Mesoscopic fluctuations

Corrections to the QPS amplitude are determined by the local values of the slipping junction parameters

$$A = \frac{A_{\mathrm{hom}}}{N} \sum_n e^{-\delta S_n - i\theta_n}$$

Mesoscopic fluctuations

Corrections to the QPS amplitude are determined
by the local values of the slipping junction parameters

$$A = \frac{A_{\rm hom}}{N} \sum_n e^{-\delta S_n - i\theta_n}$$

1. No random charges: $\theta_n = 0$

$$\langle \delta S_n \rangle = 0, \quad \langle \delta S_n \delta S_{n'} \rangle = \frac{64 E_J}{E_C} \langle \zeta_n^2 \rangle \, \delta_{nn'} \equiv \sigma^2 \delta_{nn'}$$ Gaussian, uncorrelated

$\delta S_n \ll S_{\rm hom}$ but may be $\delta S_n \gtrsim 1$ $\Rightarrow$ Strong mesoscopic fluctuations of the QPS amplitude

Central limit theorem for $N \gg e^{\sigma^2} - 1$, otherwise long tail in the distribution

Weakest junction dominates when $N \lesssim e^{0.6\, \sigma^{2/3}}$

Sum of log-normals ~ a log-normal ???

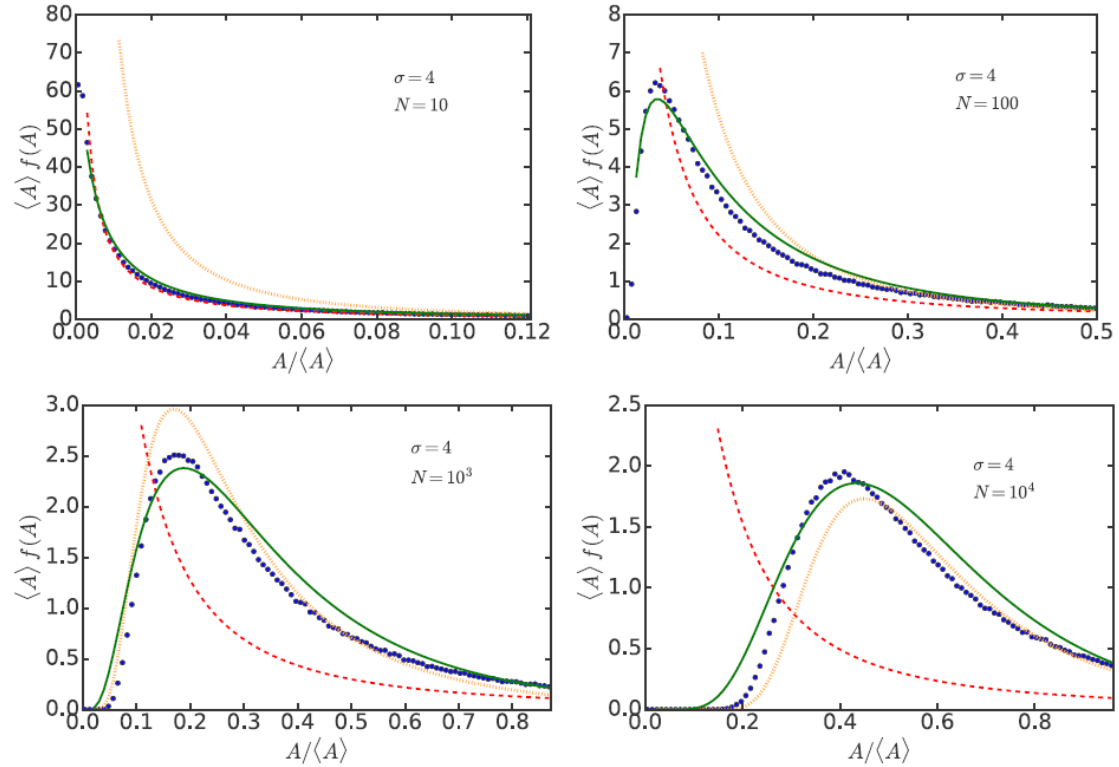
Mesoscopic fluctuations

Corrections to the QPS amplitude are determined
by the local values of the slipping junction parameters

$$A = \frac{A_{\mathrm{hom}}}{N} \sum_n e^{-\delta S_n - i\theta_n}$$

1. No random charges:

Direct numerical sampling
Saddle point approximation
Weakest junction approximation
Lognormal fit



Sum of log-normals ~ a log-normal

Mesoscopic fluctuations

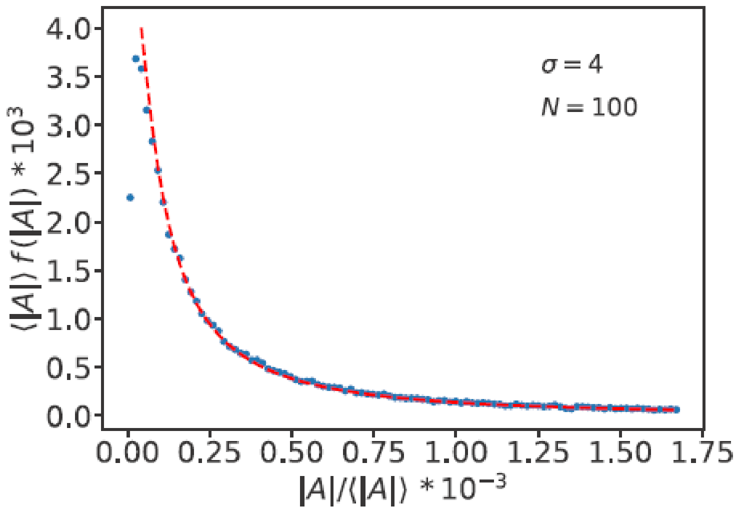
Corrections to the QPS amplitude are determined by the local values of the slipping junction parameters

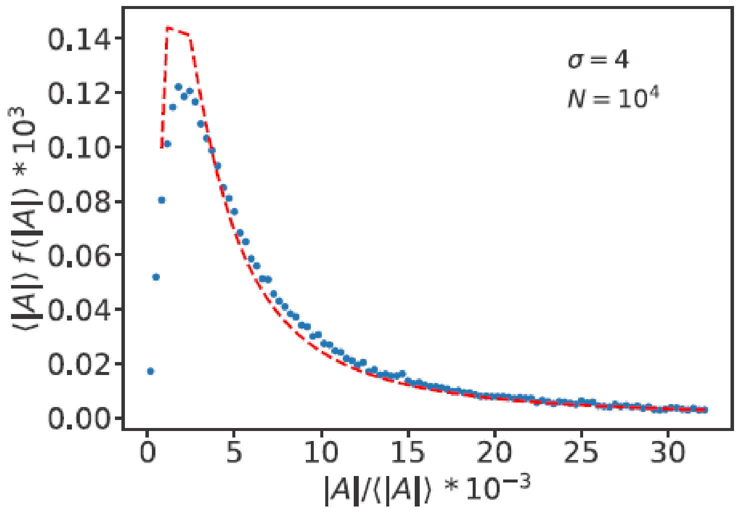
$$A = \frac{A_{\rm hom}}{N} \sum_n e^{-\delta S_n - i\theta_n}$$

2. Strong random charges: $\theta_n \in [0, 2\pi)$ uniformly and independently

Central limit theorem for $N \gg (e^{4\sigma^2} - 1)/2$

Weakest junction dominates when $N \lesssim e^{\sigma^{2/3}}$





Conclusions

Theory of coherent QPSs in spatially inhomogeneous JJ chains:

spatial inhomogeneity

$\downarrow$

Mooij-Schön modes

$\downarrow$

hydrodynamic part of the QPS action

homogeneous chain: $g \ln \frac{N}{\ell_s}$

periodic spatial modulation: $g \ln \frac{N}{\ell_s} + \delta g \ln \frac{a}{\ell_s}$

random spatial modulation: $g \ln \frac{N}{\ell_s} + \frac{\delta g}{\sqrt{\ell_s}}$

Mesoscopic fluctuations of the QPS amplitude
are dominated by the local values of the parameters