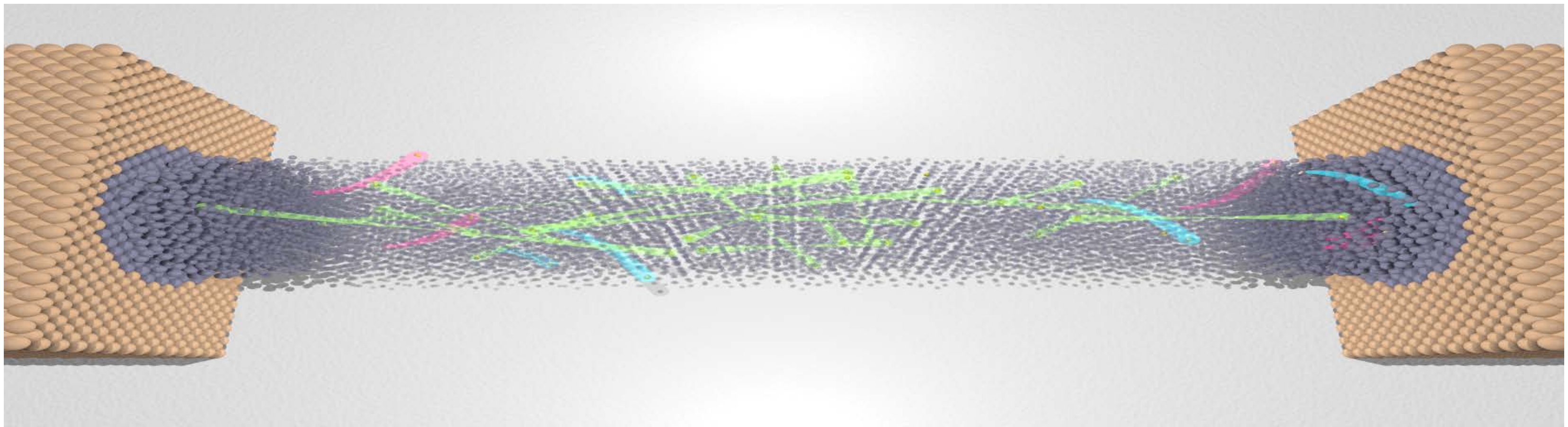
Pair-breaking quantum phase transition in superconducting nanowires

Andrey Rogachev
Department of Physics, University of Utah



Hyunjeong Kim, Frédéric Gay, Adrian Del Maestro, Benjamin Sacépé, AR, *Nature Physics* (2018).
(Institute Néel, Grenoble France, University of Vermont, University of Utah)

1. **Pair-breaking QPT in nanowires**
2. Reliability of the finite-size scaling analysis (questionable)
3. Field-induced QPT in homogeneously disordered films likely needs to be revisited.
4. Suppression of superconductivity in nanowires in zero magnetic field (still a puzzle)

5. Zero bias anomaly in nanowires (due to electron heating). H. Kim and AR, PRB (2016)

Quantum Phase Transitions

QPT – phase transition between two ground states of matter at zero temperature

QPT occur in many systems:

*** Condensed matter:**

metal – insulator, superconductor-insulator, transitions between magnetic phases.

*** Stars**

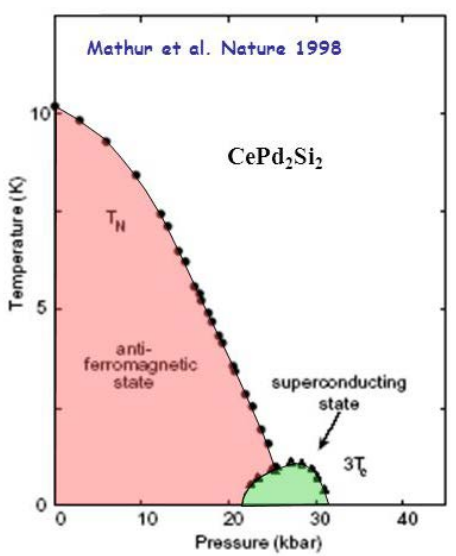
*** Dense quark matter**

*** Atomic nuclei**

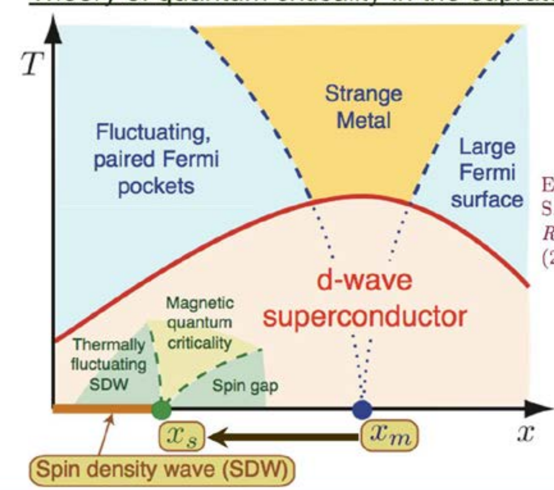
*** Cold atomic gases**

Quantum fluctuations -> new phases

heavy fermion superconductor



Theory of quantum criticality in the cuprates

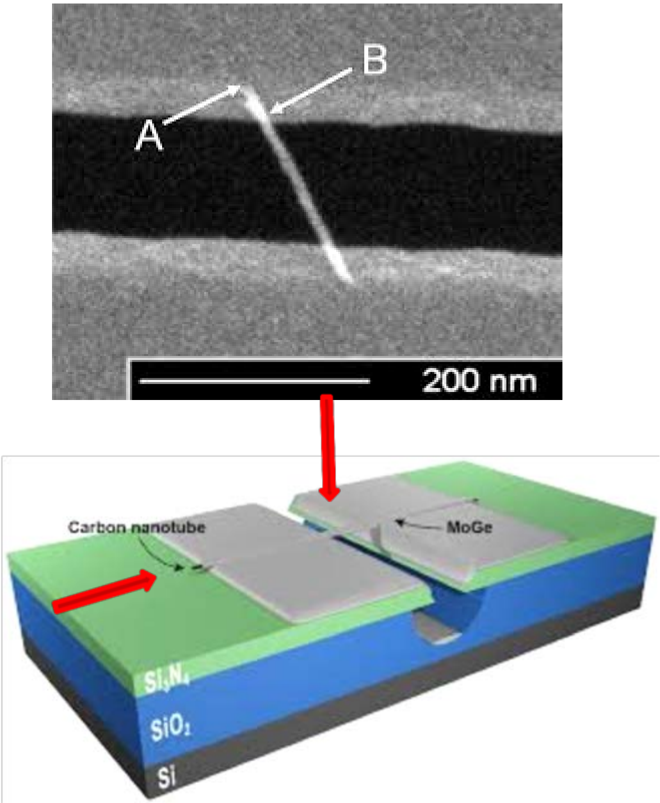


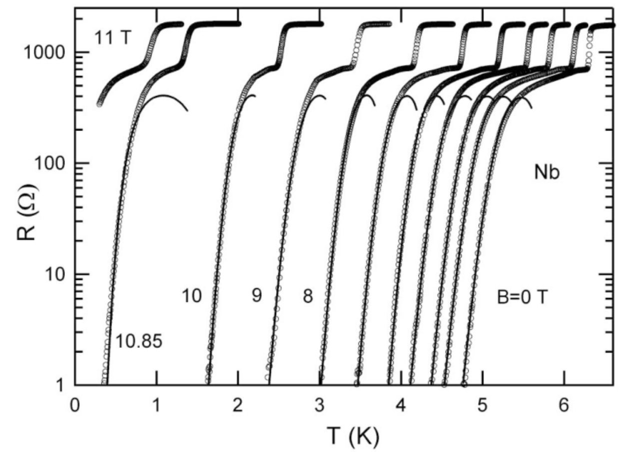
E. G. Moon and S. Sachdev, *Phy. Rev. B* **80**, 035117 (2009)

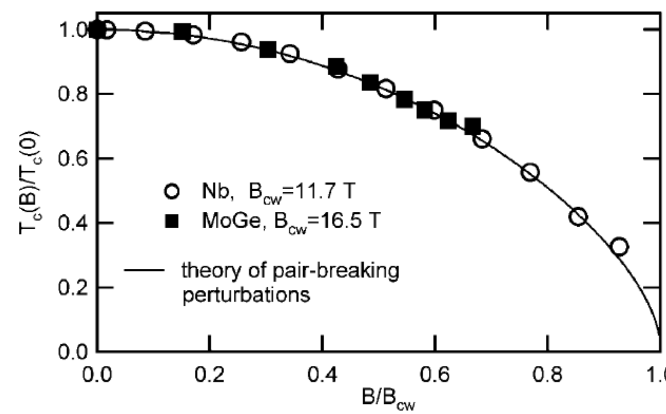
Competition between SDW order and superconductivity moves the actual quantum critical point to $x = x_s < x_m$.

The story begins: Effect of strong magnetic field on MoGe and Nb nanowires

(AR, Bollinger, Bezryadin PRL (2005))







Fabrication: molecular templating technique
Deposition on suspended carbon nanotube

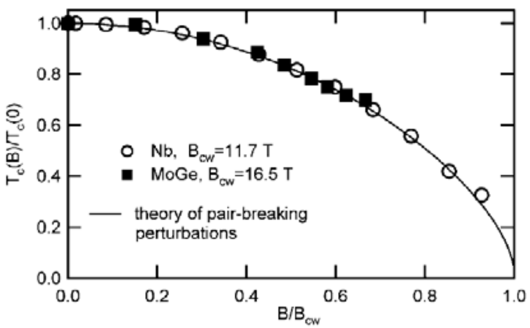
Quantitative agreement with the pair-breaking theory !!!

$Mo_{78}Ge_{22}$ Amorphous
Mean free path 0.3 nm
T_C=7 K (bulk)

Pair-breaking suppression of superconductivity

Pair-breaking by magnetic impurities
Abrikosov-Gor'kov JETP(1961)

Pair-breaking effect of magnetic field
Tinkham's book (p. 390-399)



$$\ln\left(\frac{T_C(B)}{T_{C0}}\right)=\psi\left(\frac{1}{2}\right)+\psi\left(\frac{1}{2}+\frac{\alpha(B)}{2\pi k_B T_C(B)}\right)$$

ψ di-gamma function

$$\alpha=\alpha_S+\alpha_O$$

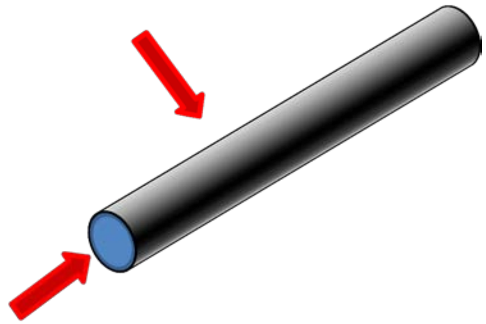
Spin and orbital pair-breaking strengths

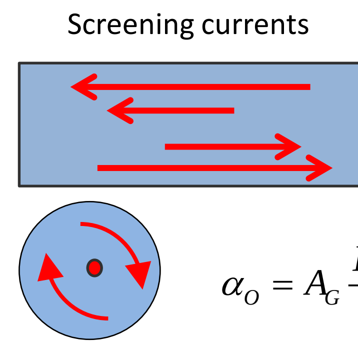
$$1.76k_B T_{C0}=2(\alpha_S(B_C)+\alpha_O(B_C))$$

Mean-field critical field

Vortices do not form in 1D superconductors $diameter<\pi\sqrt{2}\xi_{GL}(0)$

Orbital pair-breaking depends on field orientation





$$\alpha_O=A_G\frac{De^2B^2}{6\hbar}$$

$$A_G(B_{\parallel})<A(B_{\perp})$$

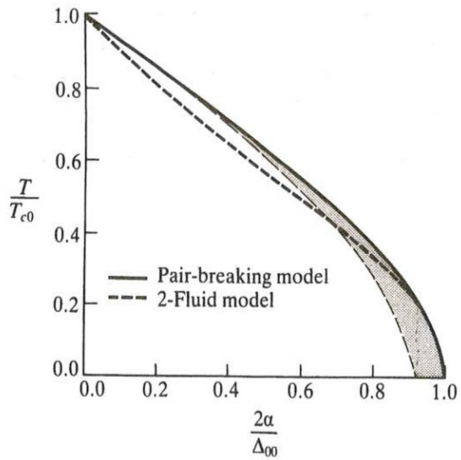
Spin pair-breaker is affected by disorder and spin-orbit scattering

$$\alpha_S=\frac{\hbar\tau_{SO}e^2B^2}{2m^2}$$

$\ell_{SO}\approx 1.5$ nm in MoGe alloys

$$\tau_{s.o.}\simeq\tau(\hbar c/Ze^2)^4$$

Important conclusions from AG pair-breaking theory



$$\alpha = \frac{DeH}{c} \quad \text{bulk type II in vortex state}$$

$$\alpha = 0.59\,\frac{DeH}{c} \quad \text{surface sheath}$$

$$\alpha = \frac{1}{6}\frac{De^2H^2d^2}{\hbar c^2} \quad \text{thin film, parallel field}$$

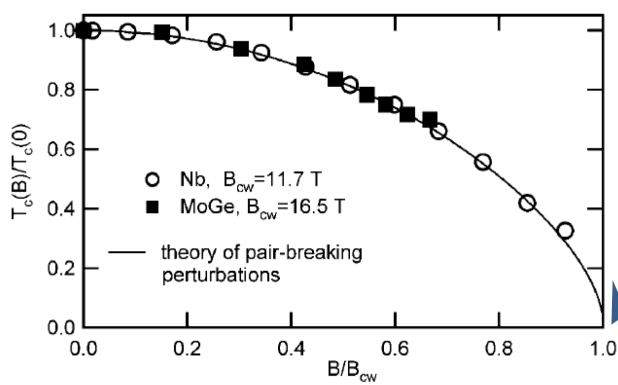
$$\alpha = \frac{DeH}{c} \quad \text{thin film, perpendicular field}$$

$$\alpha = \frac{2De^2\langle A^2\rangle}{\hbar c^2} \quad \text{small particle}$$

1) Gapless superconductivity near critical field (lots of normal electrons)

2) PB theory can be applied to films in both parallel and transverse field orientations.

Pair breaking theory does not include effects of fluctuations.
What happens near Bc and at higher fields? Do we have quantum phase transition at Bc?

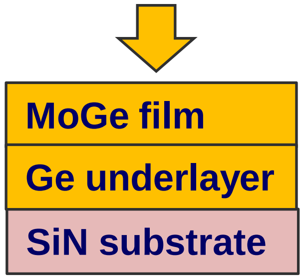


???

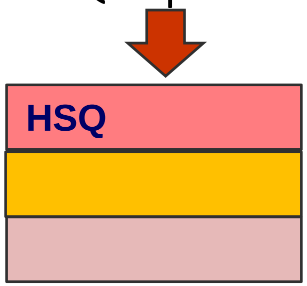
Our work on molecular templated nanowires inspired theoretical and experimental development

Our fabrication method - electron beam lithography with negative HSQ resist

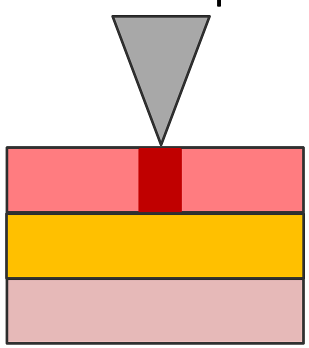
Film deposition



HSQ deposition

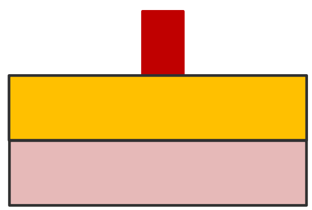


E-beam exposure



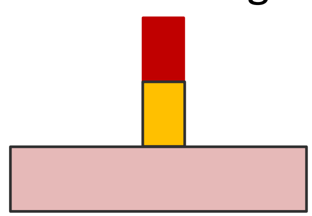
$Mo_{78}Ge_{22}$
Amorphous
Mean free path 0.3 nm
T_C=7 K (bulk)

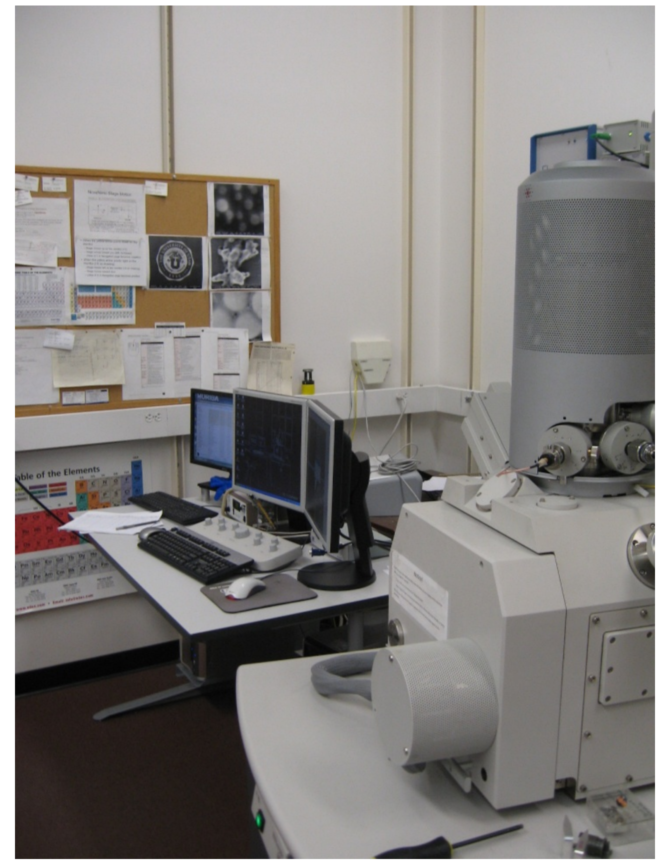
HSQ development



nanowire

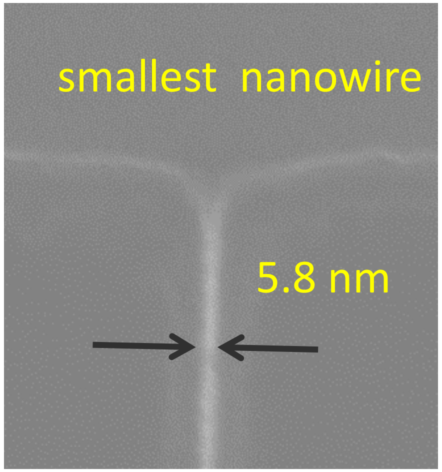
SF6 reactive
ion etching

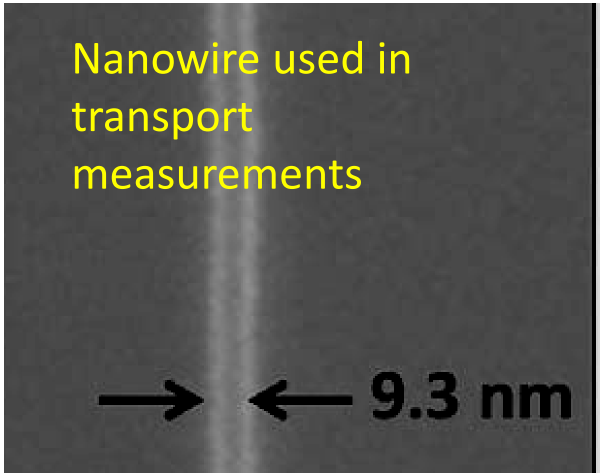


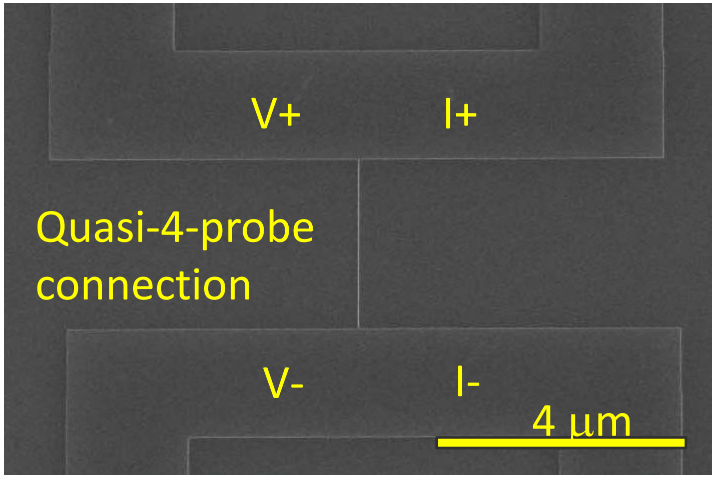


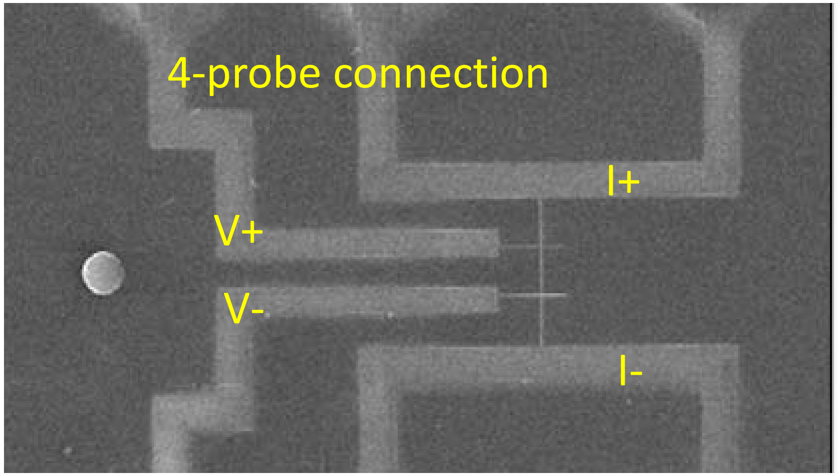
SEM - FEI Novo Nano 600
Field emission cathode

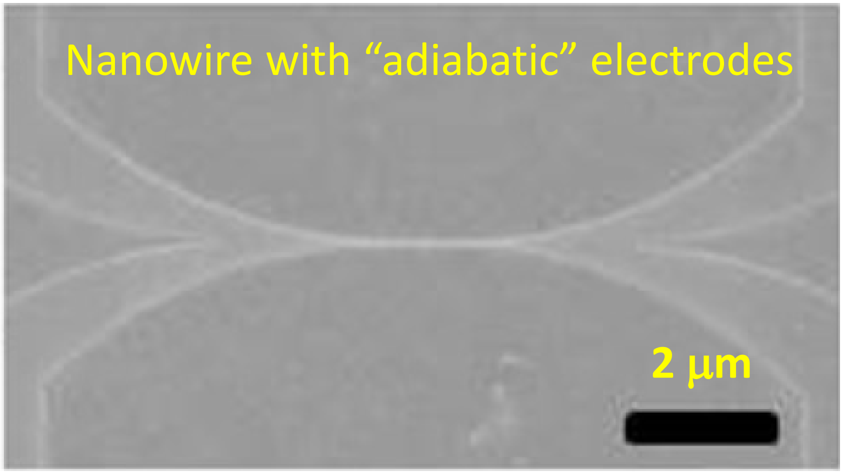
Nanowires and patterns made with EBL











Narrow lines –as good as molecular templating technique
No length limitation (well actually 120 μm)
Flexibility in pattern creation

Nanowire length range
150 nm – 25 μm
Nanowire thickness 4 – 8 nm

Theory development:

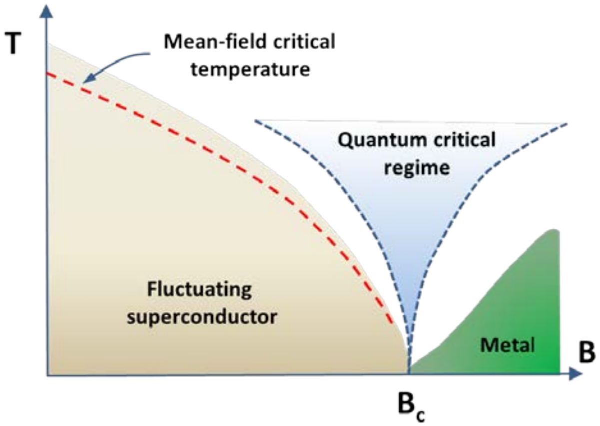
<u>Perturbative theory:</u>
N. Shah and A. Lopatin, *Microscopic analysis of the superconducting quantum critical point: Finite-temperature crossovers in transport near a pair-breaking quantum phase transition*, *Phys. Rev. B* 76, 094511 (2007).

<u>Quantum Critical Theory:</u>
Adrian Del Maestro, B. Rosenow, N. Shah, and S. Sachdev, *Universal thermal and electrical transport near the superconductor-metal quantum phase transition in nanowires*, *Phys. Rev. B* **77**, 180501(R) (2008).
Adrian Del Maestro, B. Rosenow, and S. Sachdev, *Theory of the pairbreaking superconductor–metal transition in nanowires, Annals Phys*., **324**, 523-583 (2009).

Motivation for quantum critical theory
I. Herbut, *Zero-temperature d-wave superconducting phase transition. PRL (2000)*
V. Galitski, *Nonperturbative microscopic theory of superconducting fluctuations near a quantum critical point*, *Phys. Rev. Lett.,* **100**, 127001 (2008).
(both work are on critical effect of disorder on d-wave superconductors)

Pair breaking critical theory for QPT in superconducting nanowires

A. Del Maestro, B. Rosenow, and S. Sachdev, *Annals Phys.*, **324**, 523-583 (2009).



Quantum critical action for the ohmically damped complex Cooper pair order parameter

Pair-breaking term

$$S = \int_0^L dz \int_0^{\hbar\beta} d\tau \left[D\, |\partial_z \Psi(z,\tau)|^2 + \alpha\, |\Psi(z,\tau)|^2 + \frac{u}{2}\, |\Psi(z,\tau)|^4 \right] + \frac{\eta}{\hbar\beta} \sum_{\omega_n} \int_0^L dz\, |\omega_n| |\Psi(z,\omega_n)|^2$$

Damping term

Interaction between superconducting fluctuations

$$\Psi(z,\omega_n) = \int_0^{\hbar\beta} d\tau\, \Psi(z,\tau) e^{i\omega_n \tau}$$

- Infinitely large bath on normal electrons
- Number of Cooper pairs is not conserved (important difference compared to phase-only theories, such as “dirty bosons” models or theories based on quantum phase slips)
- Superconducting fluctuations include both amplitude and phase and are Aslamazov-Larkin-type, not phase-slip-type.

Predictions of breaking critical theory

$$\sigma(T) = \frac{(e^*)^2}{\hbar}\left(\frac{\hbar D}{k_B T}\right)^{1/z} \Phi_\sigma\left(\frac{\hbar(\alpha-\alpha_c)^\nu}{k_B T^{1/z}}\right),$$

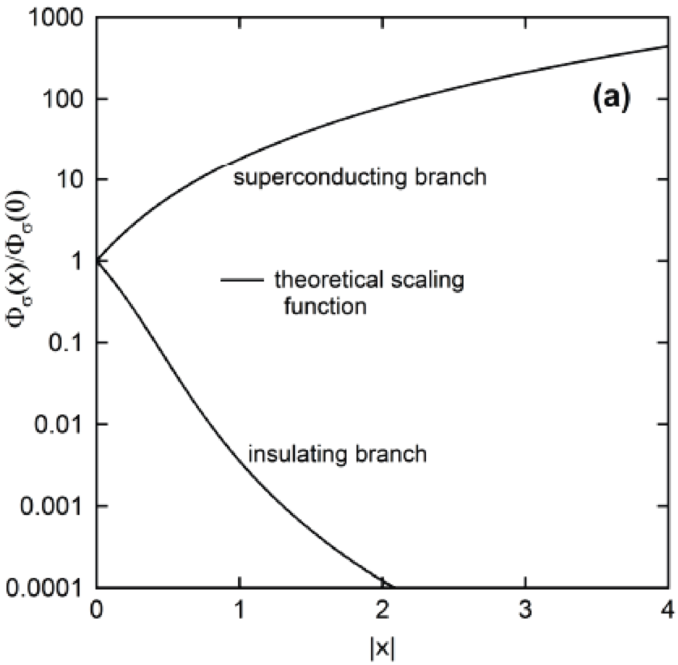
behavior of conductivity in quantum critical regime

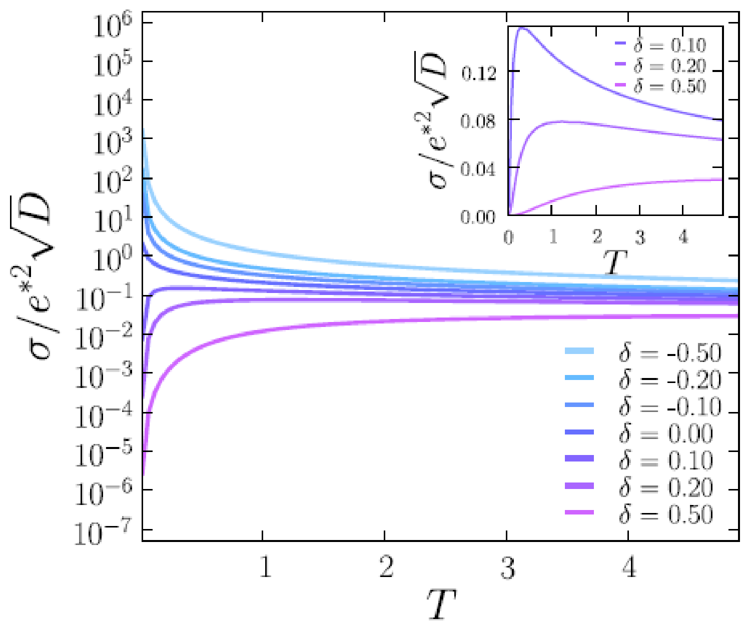
$e^* = 2e$

$z \approx 2$ in so-called "large-N" approximation, N – number of components of order parameter

$\nu \approx 1$ Indicate strong interaction between superconducting fluctuations

Theoretical scaling function was computed numerically !!!

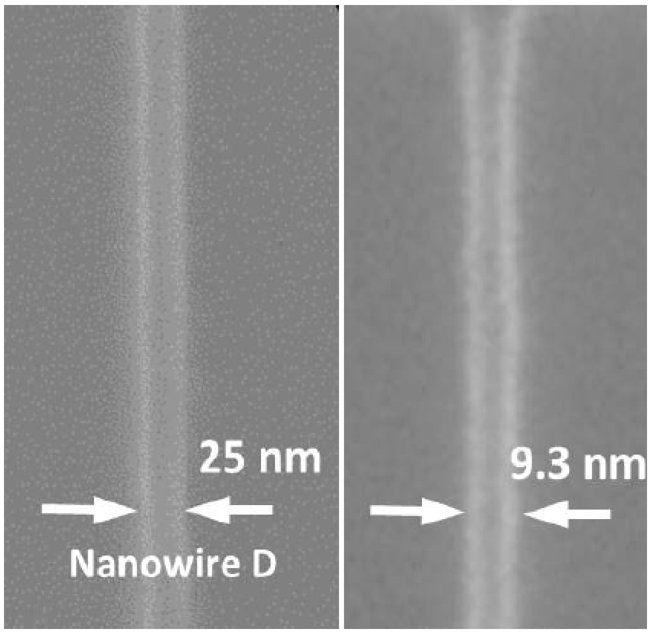




Experiment

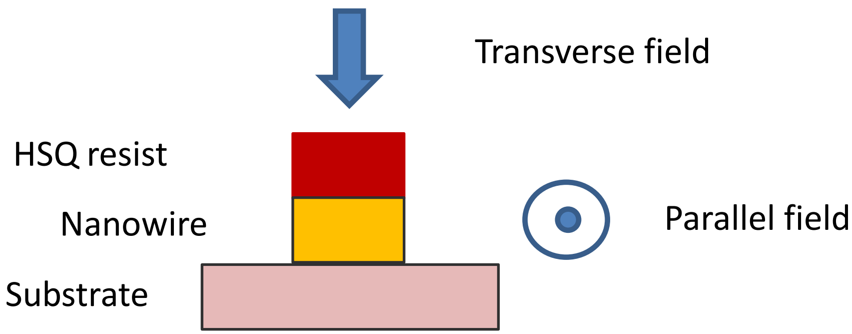
Nanowire E: $Mo_{78}Ge_{22}$, $t = 6$ nm, $w = 13$ nm, $T_c = 1.5$ K

nanowire D: $Mo_{50}Ge_{50}$, $t = 10$ nm, $w = 25$ nm, $T_c = 0.6$ K



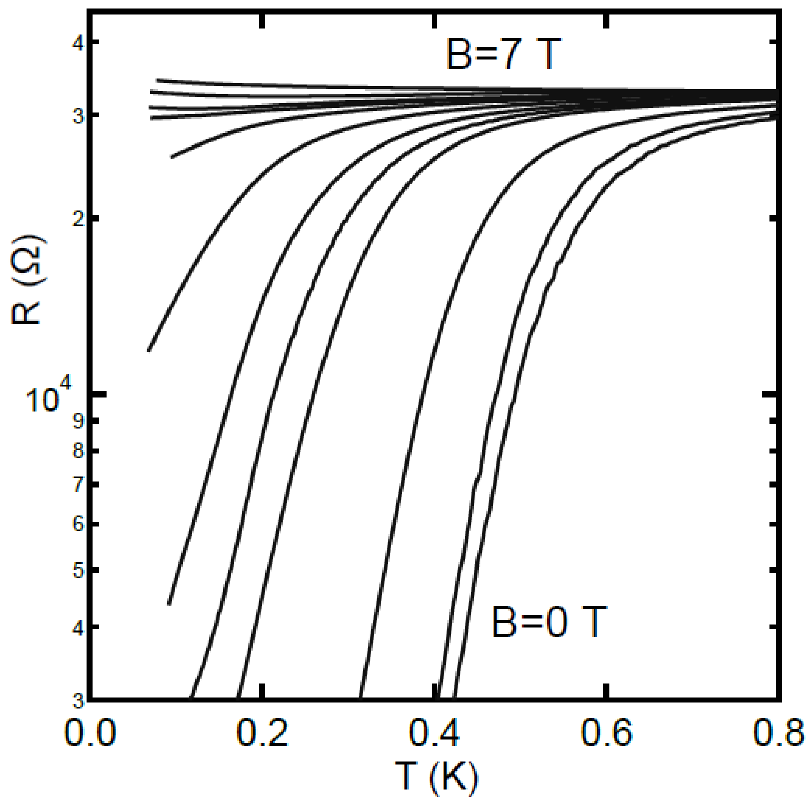
Very small variation of width ±0.7 nm

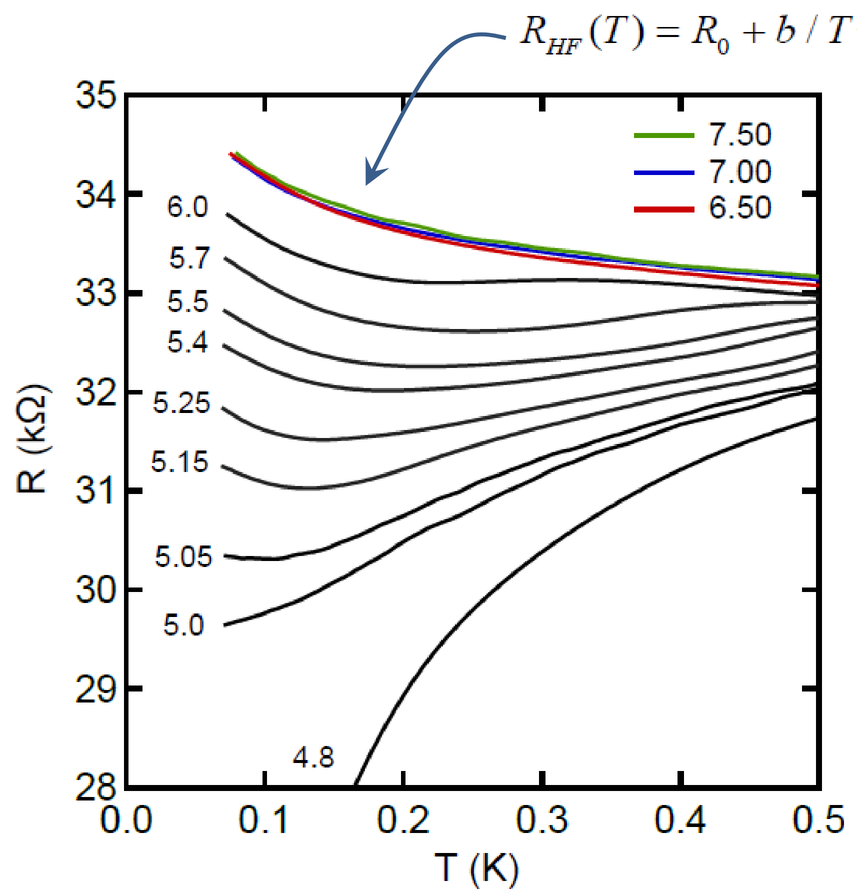
Nanowire has roughly rectangular cross section



Measurements are done in dilution refrigerator with well-filtered lines in transverse and parallel magnetic fields

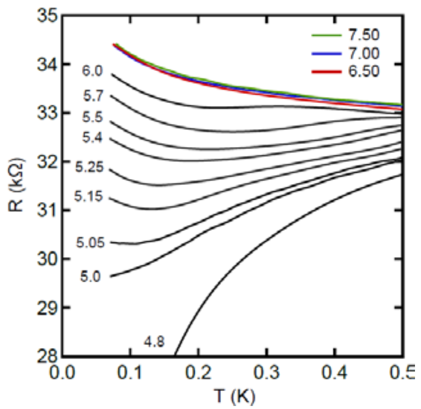
Nanowire D in parallel magnetic field





- Re-entrant behavior
- At high-field R(T) curves do not depend on magnetic field

Comparison with the theory



1) QPT takes place ***only in superconducting part*** of the system. We need to extract it.

Two fluid model $G_{SC} = G_{EXP} - G_{NORN}$ $\quad G_{NORM} = G_{EXP}\,(\text{High Field})$

Both Drude and Quantum Correction terms are included

$$\sigma(\delta, T, E) = \frac{Q^2}{\hbar}\left(\frac{k_B T}{\hbar c}\right)^{(d-2)/z} \Phi_\sigma\left(\frac{\delta}{T^{1/z\nu}}, \frac{\delta}{E^{1/\nu(z+1)}}\right)$$

2) At the critical field the conductance varies as $\quad G_{SC} \sim T^{(d-2)/z} \approx 1/T^{0.5} \quad (z \approx 2)$

3) Pair breaker strength varies with field as $\quad \alpha \propto B^2$

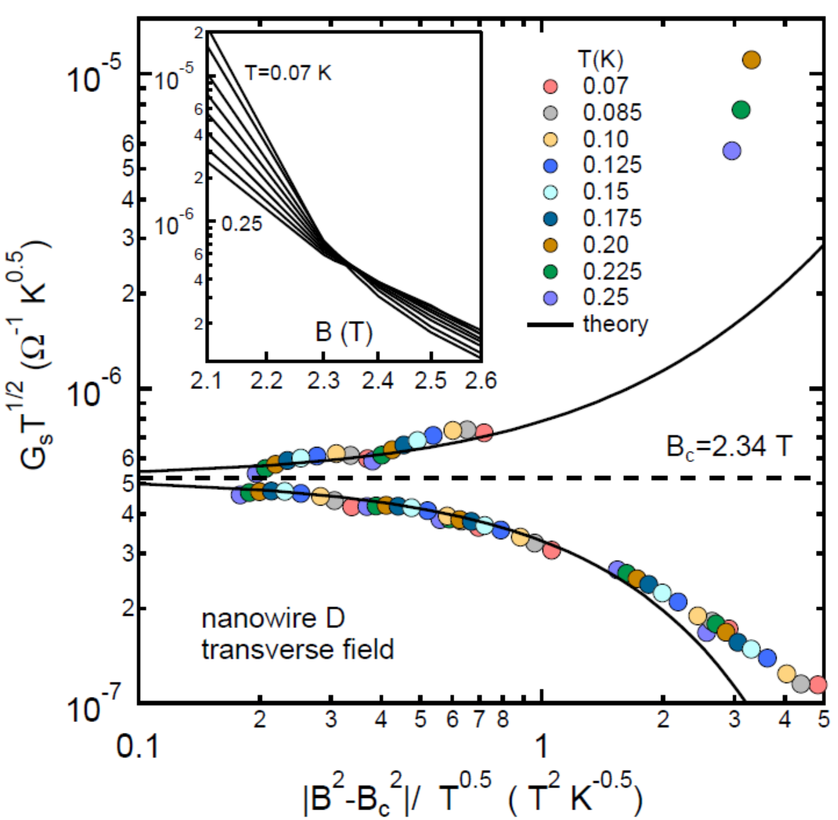
$$\sigma(T) = \frac{(e^*)^2}{\hbar}\left(\frac{\hbar D}{k_B T}\right)^{1/z} \Phi_\sigma\left(\frac{\hbar(\alpha - \alpha_C)^\nu}{k_B T^{1/z}}\right), \qquad G_{SC} = \frac{4e^2}{\hbar}\left(\frac{\hbar D}{k_B T}\right)^{1/2} \Phi_\sigma\left(A \times \frac{\left|B^2 - B_c^2\right|}{T^{1/2}}\right) \qquad \begin{array}{c}(\nu \approx 1)\\ (z \approx 2)\end{array}$$

The finite-size scaling

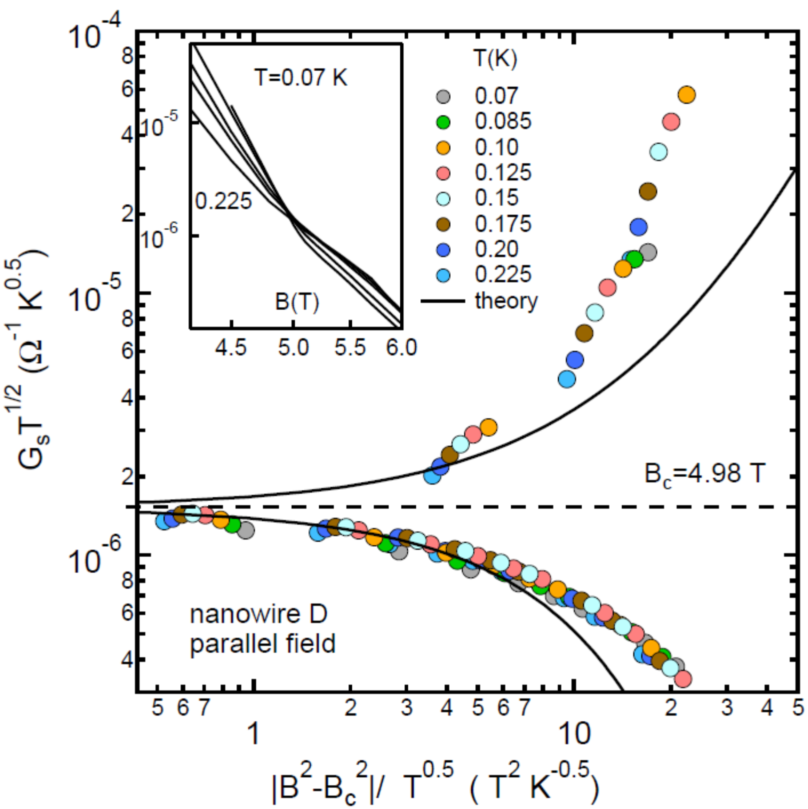
$$G_{SC} = \frac{4e^2}{\hbar}\left(\frac{\hbar D}{k_B T}\right)^{1/2} \Phi_\sigma\left(A \times \frac{\left|B^2 - B_c^2\right|}{T^{1/2}}\right)$$

nanowire D: $Mo_{50}Ge_{50}$, $t = 10$ nm, $w = 25$ nm, $T_c = 0.6$ K

Transverse field



Parallel field

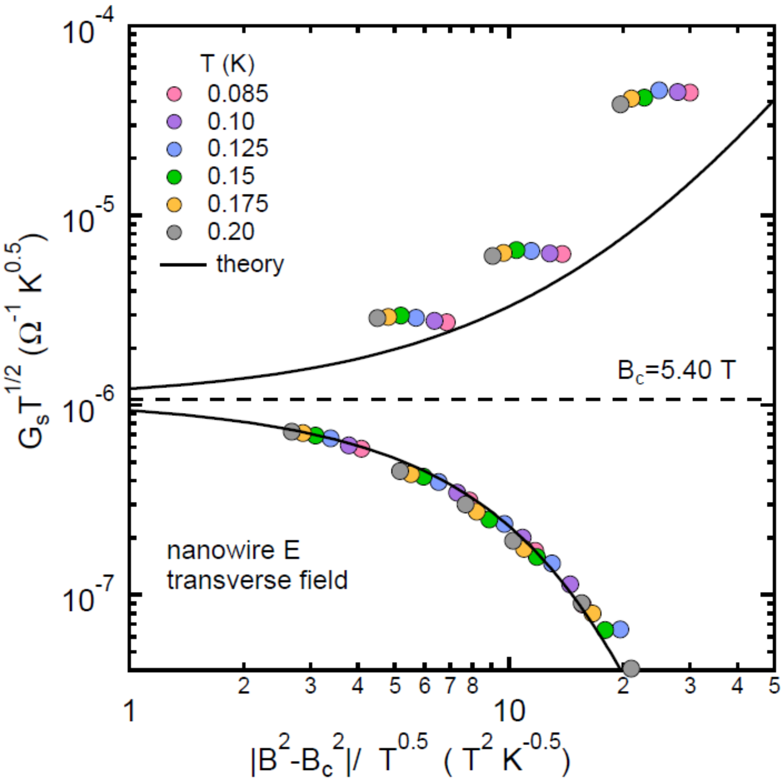


The scaling

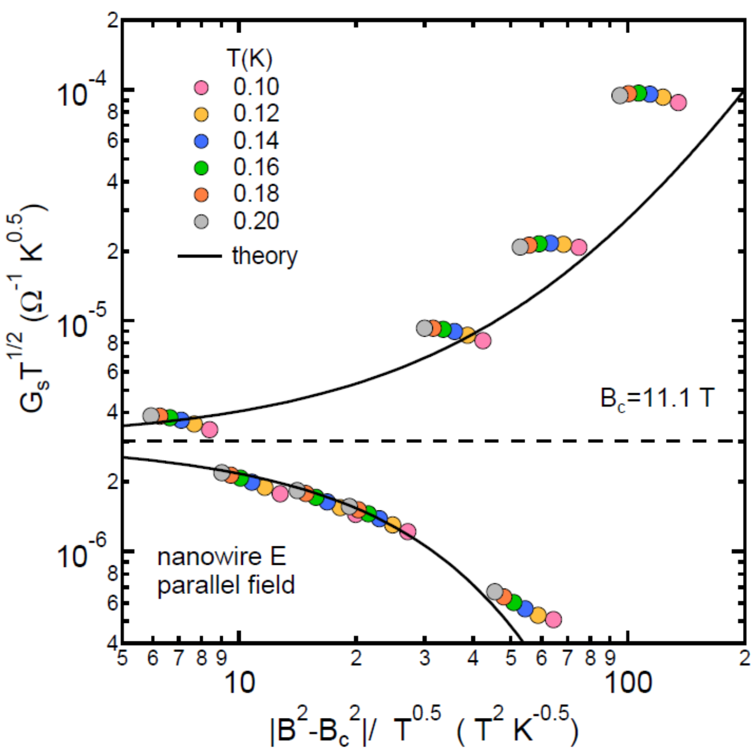
$$G_{SC} = \frac{4e^2}{\hbar}\left(\frac{\hbar D}{k_B T}\right)^{1/2} \Phi_\sigma\left(A \times \frac{|B^2 - B_c^2|}{T^{1/2}}\right)$$

Nanowire E: $Mo_{78}Ge_{22}$, $t = 6$ nm, $w = 13$ nm, $T_c = 1.5$ K

Transverse field



Parallel field



No clear crossing in curves $G_{SC}T^{1/2}$ vs $\frac{|B^2 - B_c^2|}{T^{1/2}}$

Quantitative Comparison with the theory

$$G_{SC} = \frac{4e^2}{\hbar}\left(\frac{\hbar D}{k_B T}\right)^{1/2} \Phi_\sigma\left(A \times \frac{\left|B^2 - B_c^2\right|}{T^{1/2}}\right)$$

$$\Phi_\sigma(0) = 0.218$$

Prediction of the theory

	E (pl)	E (tr)	D (pl)	D (tr)
	$Mo_{78}Ge_{22}$	$Mo_{78}Ge_{22}$	$Mo_{50}Ge_{50}$	$Mo_{50}Ge_{50}$
B_C (T)	11.1	5.40	4.98	2.34
B_{MF} (T)	14.2	7.0	5.4	2.4
$\xi(0)$ (nm)	10	10	15	15
$(G_S\sqrt{T})_C$ $(\Omega^{-1}K^{1/2})$	2.8×10^{-6}	1.1×10^{-6}	1.55×10^{-6}	0.5×10^{-6}
$\Phi_\sigma(0)_{EXP}$	0.46	0.16	0.24	0.085

At the critical field normal electrons accounts for 80-90 % of conductance

Quantitative Comparison with the theory

Phenomenological parameters of the critical field theory can be related to microscopic parameters of the alloys via standard BSC and GL relations
Time-dependent GL theory is exact in the range of interest.

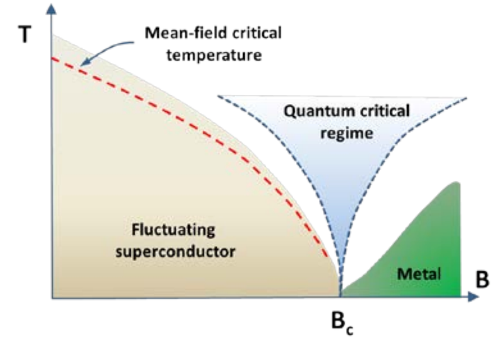
Action in the pair-breaking critical theory

$$S = \int_0^L dz \int_0^{\hbar\beta} d\tau \left[D\, |\partial_z \Psi(z,\tau)|^2 + \alpha\, |\Psi(z,\tau)|^2 + \frac{u}{2} |\Psi(z,\tau)|^4 \right] + \frac{\eta}{\hbar\beta} \sum_{\omega_n} \int_0^L dz\, |\omega_n| |\Psi(z,\omega_n)|^2$$

$$\eta = \frac{\pi^2\hbar^2}{2\,m\,\xi^2(0)} \frac{1}{8\,k_B T_c} \simeq \frac{1.5}{k_F \ell}$$
$$b = \frac{\hbar^2}{m\,\xi^2(0) n} \frac{1}{\chi(0.882\xi_o/\ell)}$$
$$u = \frac{b}{A\,\hbar^2\eta^2} \simeq 1.33 \frac{e^2 D}{\sigma_{3D} A\,\hbar^2}$$

Tucker, Halperin PRB 1971

$$\Phi_\sigma(x) = \Phi_\sigma\left(C \times 0.54 \left(\frac{\hbar k_B}{D}\right)^{1/2} (k_F \ell)^{1/2} \frac{A\sigma_{3d} T_c}{e^2} \frac{\left(B^2 - B_c^2\right)}{B_c^2 T^{1/2}} \right),$$



σ_{3d} bulk conductivity

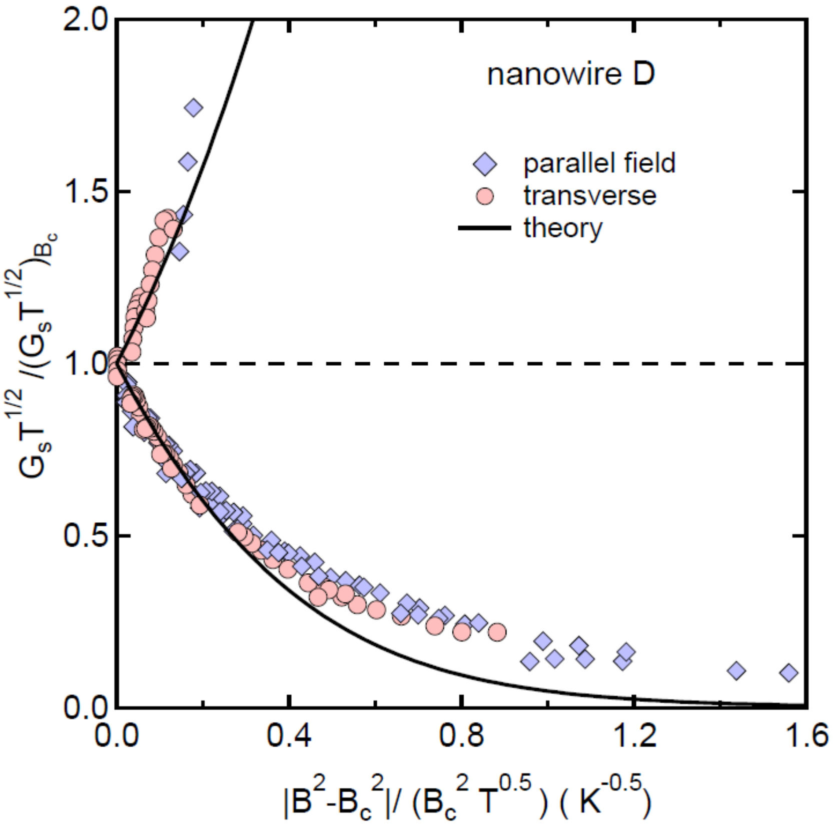
A cross sectional area

C the only adjustable parameter

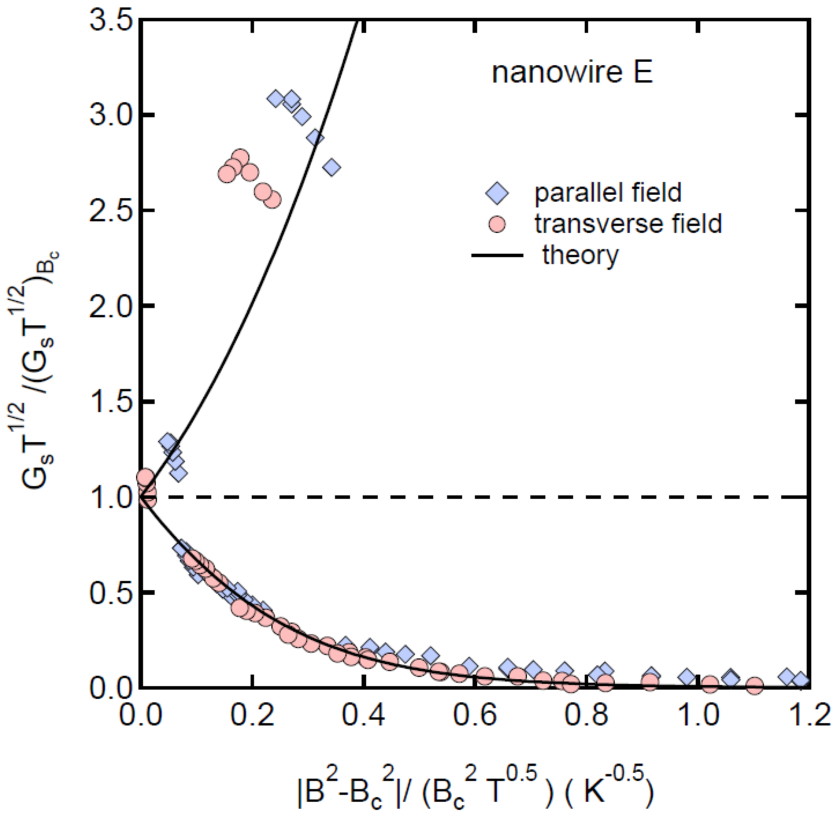
Normalization by the critical field should bring the data for different field orientation in coincidence

Quantitative Comparison with the theory

$$\sigma(T) = \frac{(e^*)^2}{\hbar}\left(\frac{\hbar D}{k_B T}\right)^{1/z} \Phi_\sigma\left(\frac{\hbar(\alpha-\alpha_C)^\nu}{k_B T^{1/z}}\right), \qquad \Phi_\sigma(x) = \Phi_\sigma\left(C\times 0.54\left(\frac{\hbar k_B}{D}\right)^{1/2}(k_F \ell)^{1/2}\frac{A\sigma_{3d}T_c}{e^2}\frac{(B^2-B_c^2)}{B_c^2 T^{1/2}}\right),$$



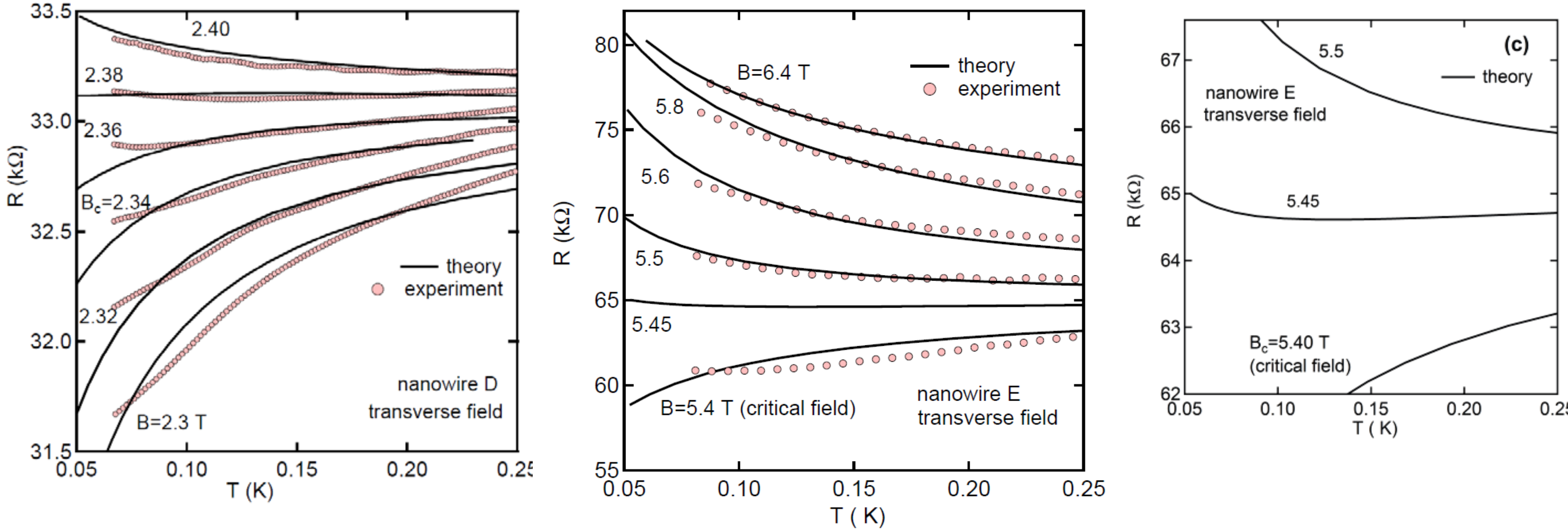
$C \approx 0.04$



$C \approx 0.05$

Reverse comparison with the theory

$$R_{th}(T,B) = \left(G_{Norm} + G_{SC}\right)^{-1}$$



Whole set of theoretical curves is generated with no extra adjusted parameters

Re-entrant behavior does not have any special significance.

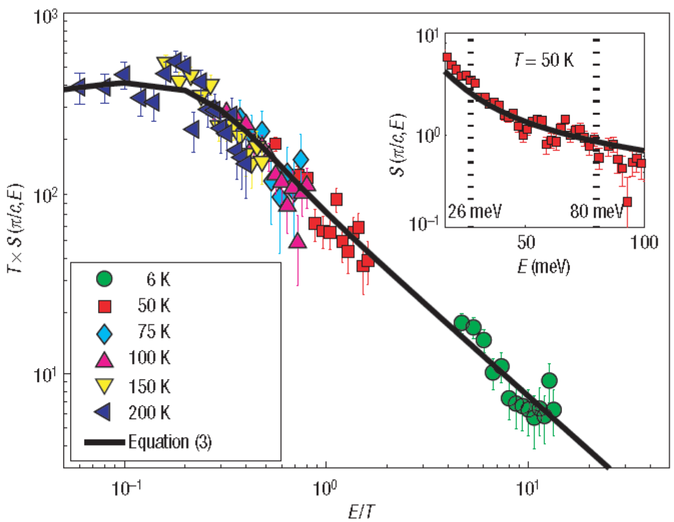
What makes QPT in nanowires an extraordinary observation.

Complete description: From microscopic parameters to long-range behavior at QPT

Heisenberg AF ½ spin chains and ladders

Bethe ansatz, Luttinger liquid,
Density Matrix Renormalization Group

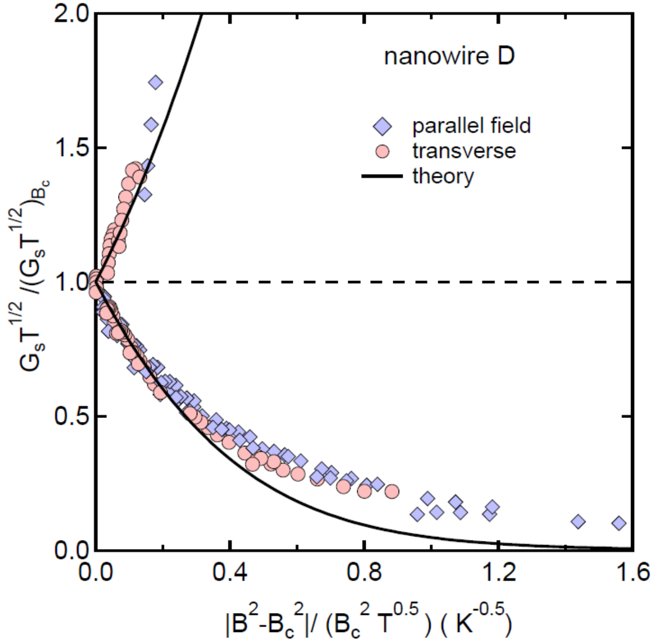
Material: $KCuF_3$ weakly coupled chains,
Inelastic neutron scattering



E,T scaling, (Lake *et al*, Nature Mat 2005**)**

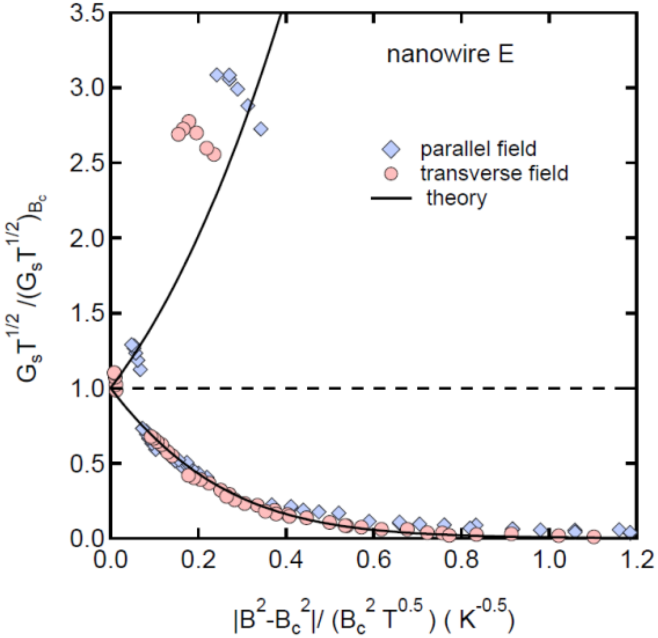
$(C_7H_{10}N)_2CuBr_4$ ladders (Povarov et al PRB 2015)

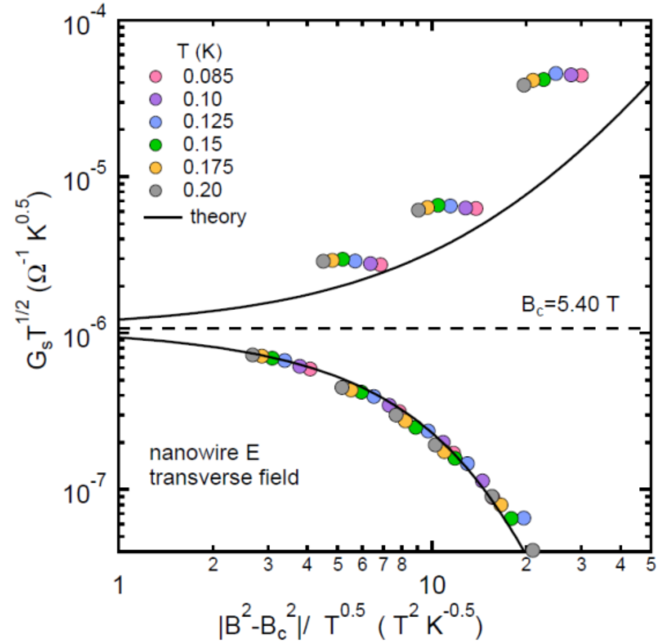
Superconducting nanowires



δ, T Scaling

Deviations from the theory, understood and not





Deviations from the theory increase near critical field and in superconducting range.
Very likely reason is not accurate procedure to extract G_{SC} $\quad G_{SC} = G_{EXP} - G_{NORN}$

$C \approx 0.04 - 0.05$??? The adjusted parameter seems too small.

$$\Phi_\sigma(x) = \Phi_\sigma\left(C \times 0.54\left(\frac{\hbar k_B}{D}\right)^{1/2}(k_F\ell)^{1/2}\frac{A\sigma_{3d}T_c}{e^2}\frac{\left(B^2-B_c^2\right)}{B_c^2T^{1/2}}\right),$$

Holographic quantum matter

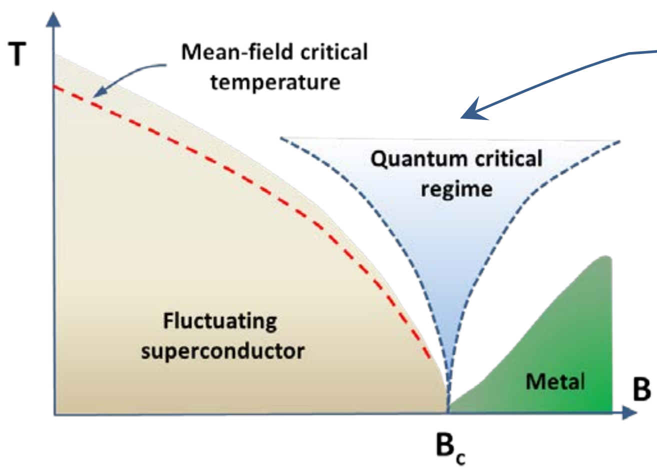
Sean A. Hartnoll[♭], Andrew Lucas[♭,♯] and Subir Sachdev[♯,△]

arXiv:1612.07324v1 [hep-th]

This review is about a particular interface between condensed matter physics, gravitational physics and string and quantum field theory. The defining feature of this interface is that it is made possible by holographic duality, to be introduced briefly in this first section. This interface has been called 'AdS/CMT'

For the condensed matter physicist: *AdS/CMT is the study of condensed matter systems without quasiparticles. In particular, AdS/CMT provides a class of models without an underlying quasiparticle description in which controlled computations can nonetheless be performed.*

AdS: anti-de-Sitter spaces in string theory
CFT: conformal field theories
CMT: condensed matter theories



$$\sigma(\delta,T,E)=\frac{Q^2}{\hbar}\left(\frac{k_BT}{\hbar c}\right)^{(d-2)/z}\Phi_\sigma\left(\frac{\delta}{T^{1/z\nu}},\frac{\delta}{E^{1/\nu(z+1)}}\right)$$

$$\sigma\sim T^{(d-2)/z}$$

The prefactor introduced from general considerations for the dynamic conductivity in the critical regime.

The same dependence is obtained from a class of holographic models using gauge-gravity duality (details are in the review)

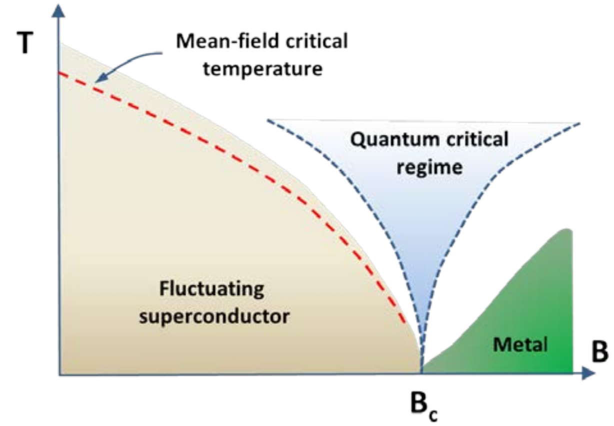
Conclusions. Summary

QPT in nanowires indeed takes place

Microscopic processes governing the transition are captured by the theory

- Pair breaking effect of magnetic field
- Interaction between Cooper pairs/Superconducting Fluctuations
- Overdamping of Cooper pairs by electronic degrees of freedom

The theory fully explains the evolution of conductivity in quantum critical regime.



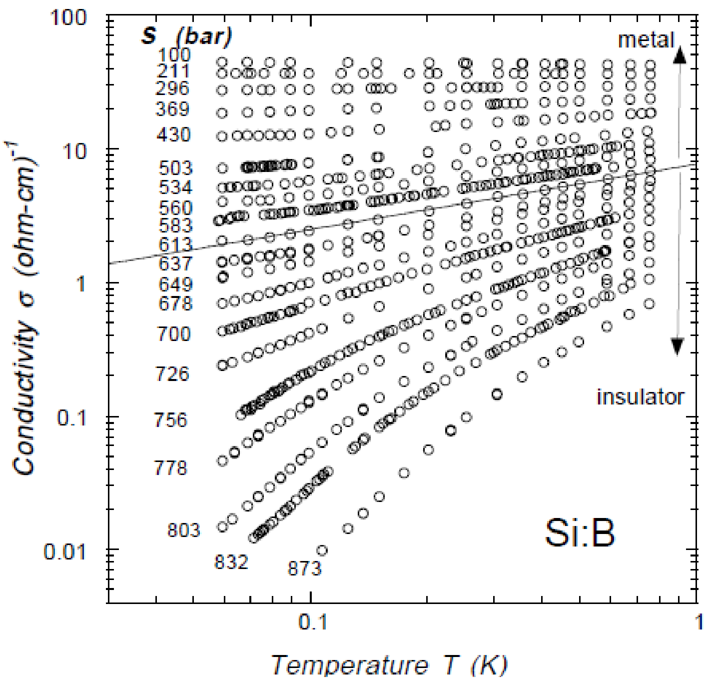
Part 2. Is the data scaling collapse a reliable indicator of QPT?

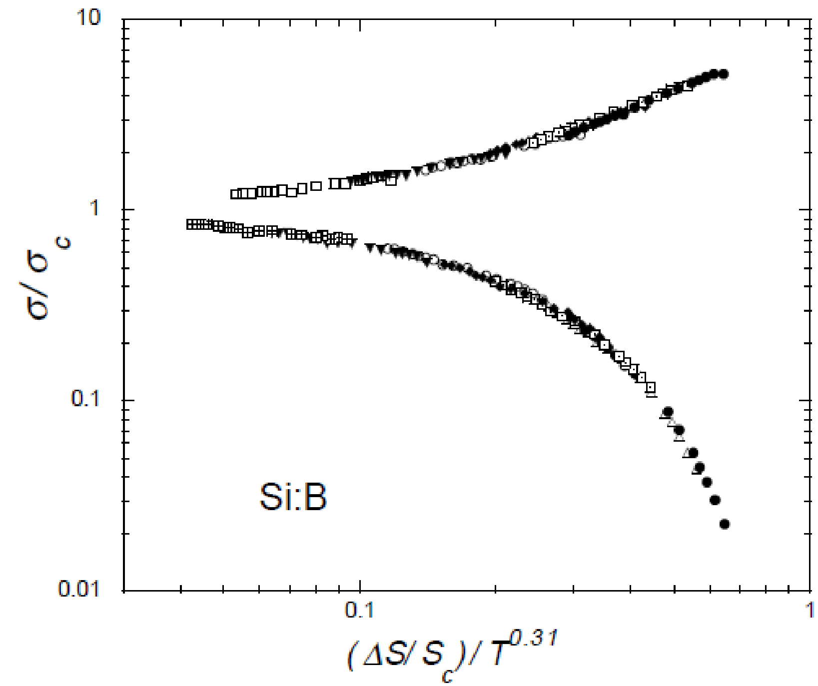
Phenomenological finite-size scaling analysis

$$\sigma(\delta,T,E)=\frac{Q^2}{\hbar}\left(\frac{k_BT}{\hbar c}\right)^{(d-2)/z}\Phi_\sigma\left(\frac{\delta}{T^{1/z\nu}},\frac{\delta}{E^{1/\nu(z+1)}}\right)$$

Metal-insulator transition (Bogdanovich, PRL 1999)

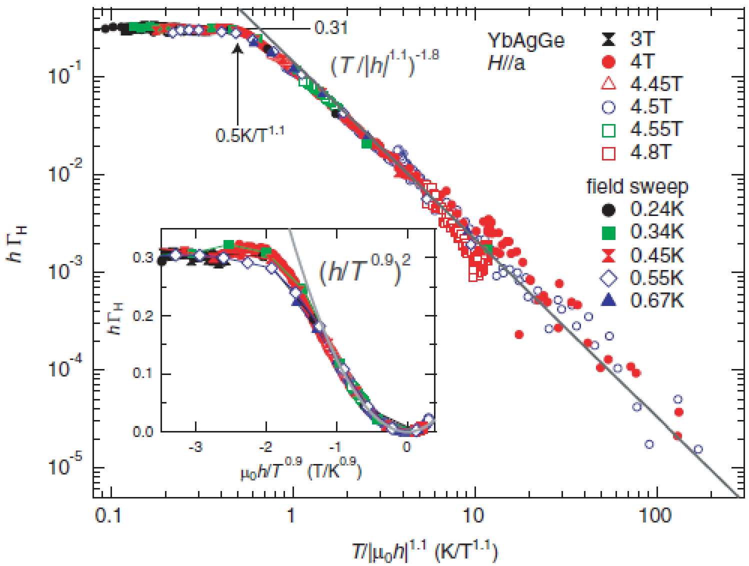
$$\sigma(t,T)=\sigma_c(T)f[(t-t_c)/T^y],\quad \sigma_c(T)=AT^x$$





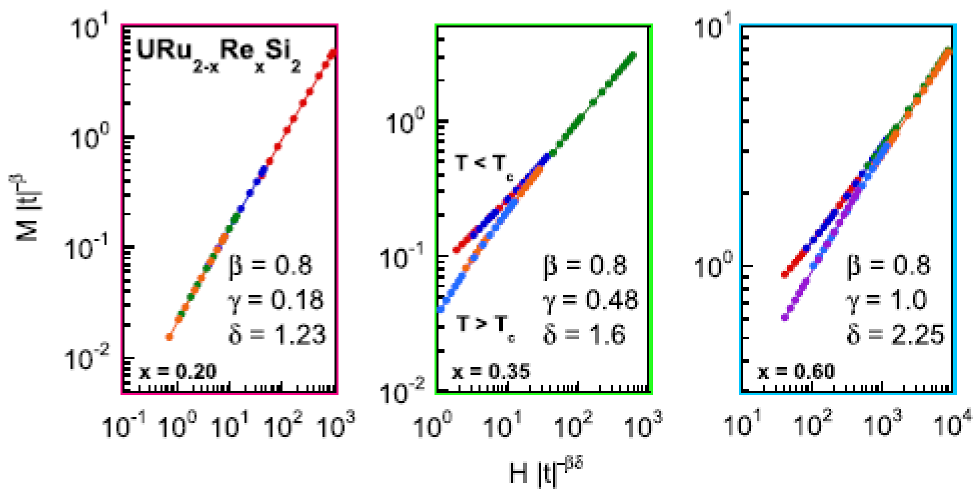
Scaling in magnetic systems

Magnetic Gruneisen parameter
(Tokiwa, PRL 2013)



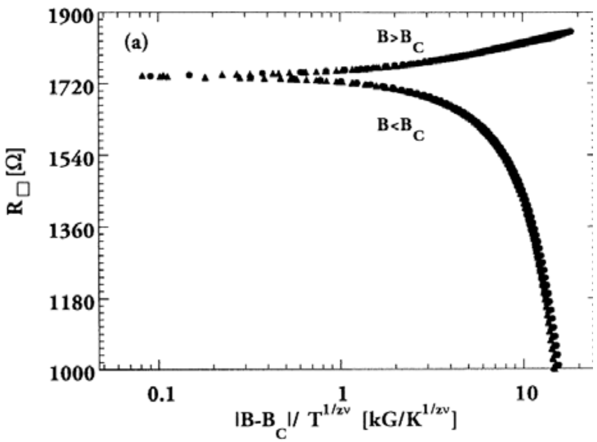
$$(\partial S/\partial H)_T = -\Gamma_H / C_H$$

Magnetization near Ferromagnetic Critical point
(Butch, PRL 2009)



Finite-size scaling analysis in superconducting films

a-MoGe films, Kapitulnik group

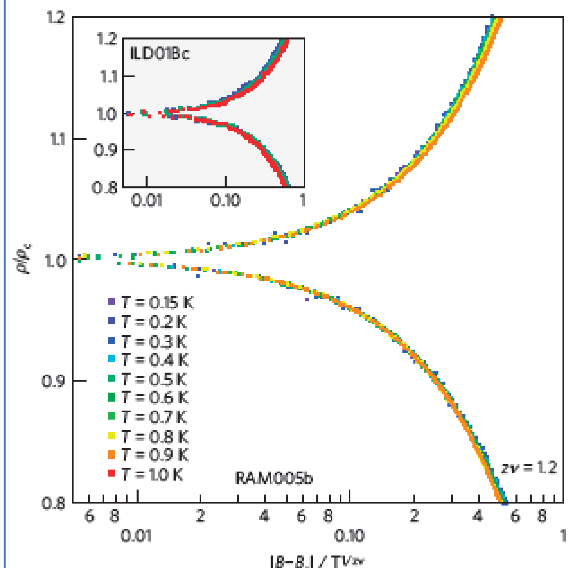


$$T = 80 - 110 \ \text{mK}$$

$$z\nu = 1.3$$

Exponents – 4/3 Classical 2D percolation theory

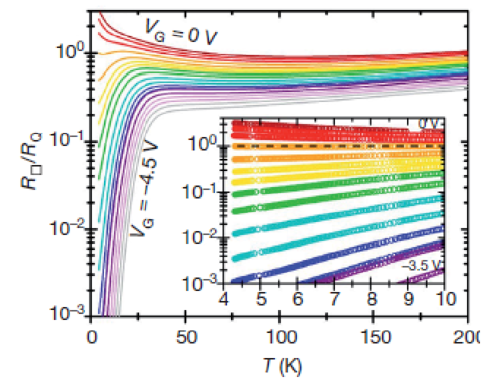
a-InO films, Shahar group

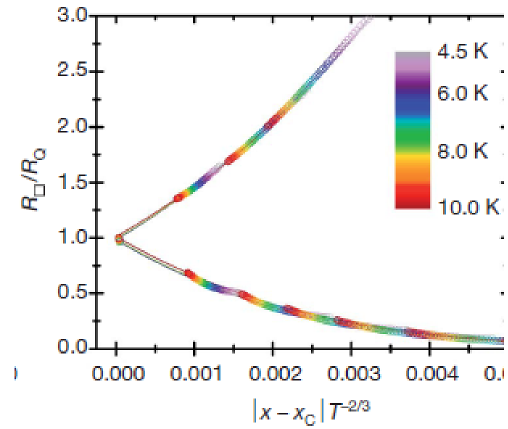


$$z\nu = 1.2$$

$$T = 150 - 1000 \ \text{mK}$$

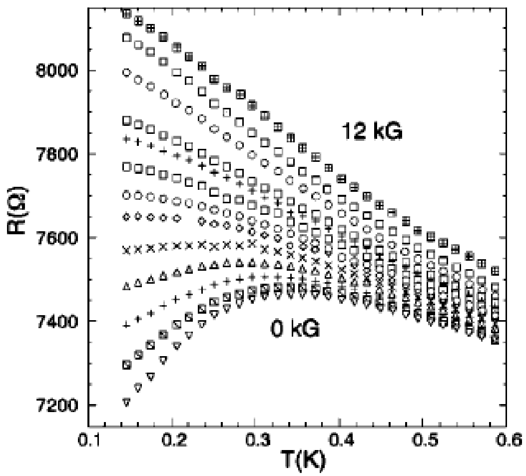
Electrostatically doped LSCO films (Božović group)

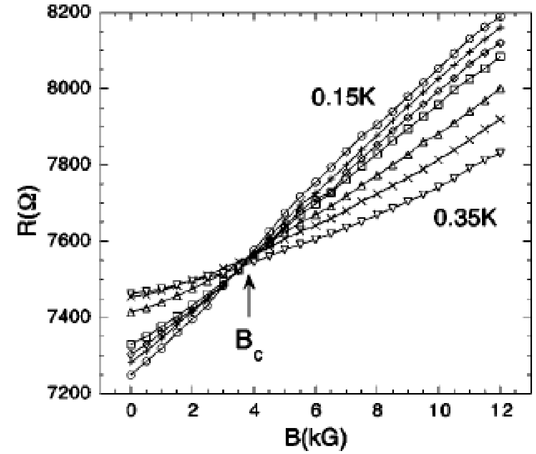




Finite-size scaling analysis in superconducting films

a-Bi, Goldman group





Crossing point implies Discontinuous SIT

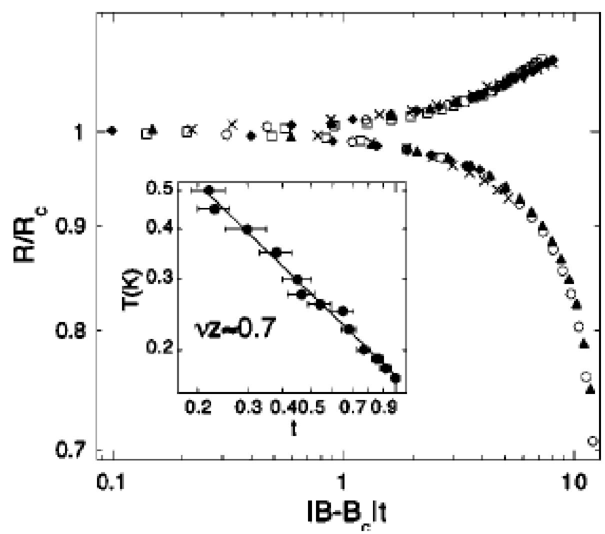
$$R(T,B) = R_c F(|B - B_c| / T^{1/z\nu})$$

Scaling equation

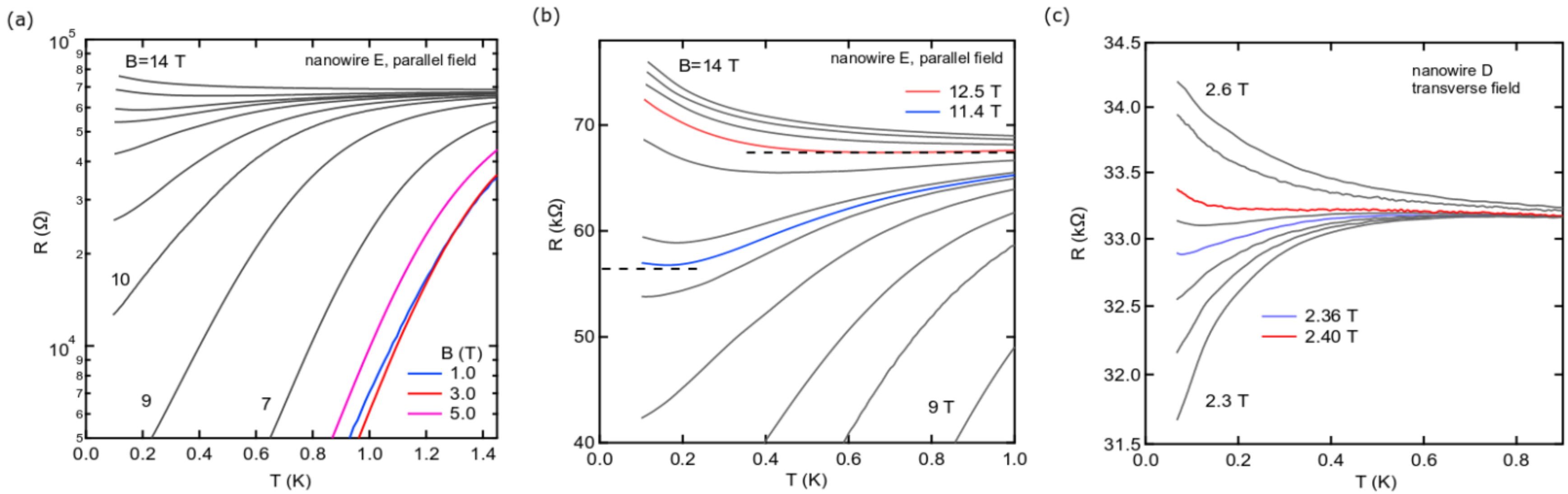
$z\nu = 1.4$ Thickness-tuned transition

$z\nu = 0.7$ B-tuned transition

$$\sigma(\delta,T,E) = \frac{Q^2}{\hbar}\left(\frac{k_B T}{\hbar c}\right)^{(d-2)/z} \Phi_\sigma\left(\frac{\delta}{T^{1/z\nu}}, \frac{\delta}{E^{1/\nu(z+1)}}\right)$$



Data for nanowires look very much like data for films



$$R(B,T) = R_c \Phi_R \left(\frac{|B - B_c|}{T^{1/z\nu}} \right)$$

Phenomenological scaling equation based on "dirty boson" model

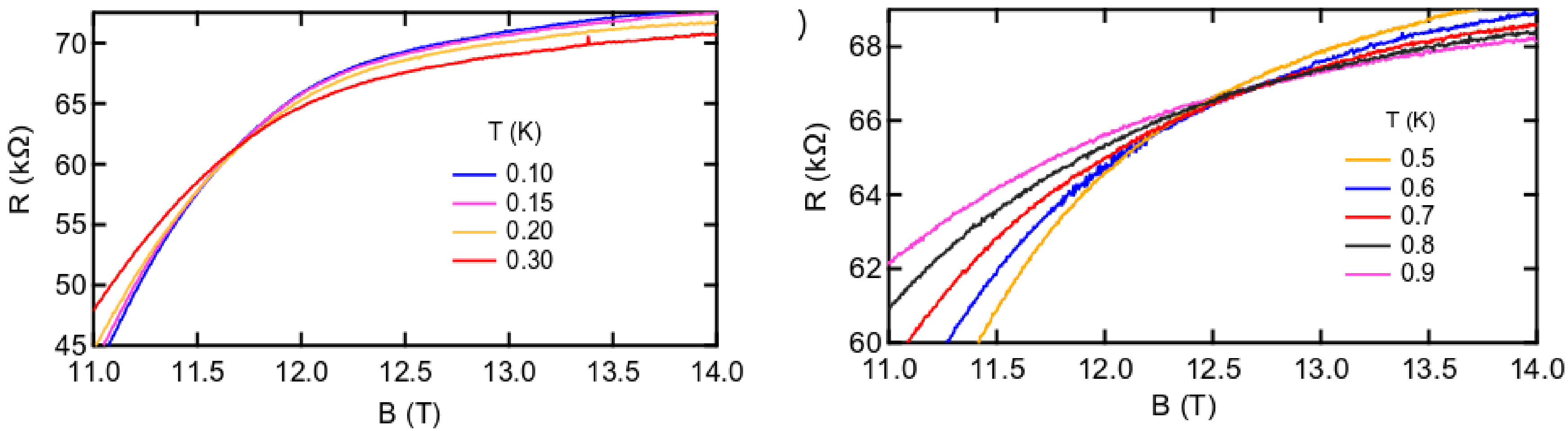
We want to use the scaling equation in the form suitable for 2d systems to scale the data for our 1d nanowires

---

Two "flat" sections were taken as an indication of two critical fields corresponding two consequent QPS

$LaAlO_3/LaTiO_3$ interface superconductors, Biscaras et al Nat Mat (2013**)**
Underdoped $La_{2-x}Sr_xCuO_4$ X. Shi, Nat Phys (2014)

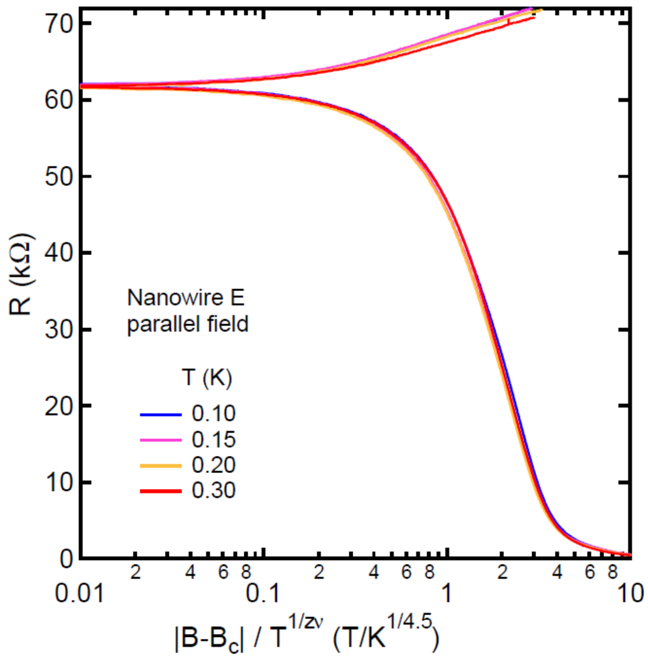
2d scaling analysis for 1d nanowires: curve crossing

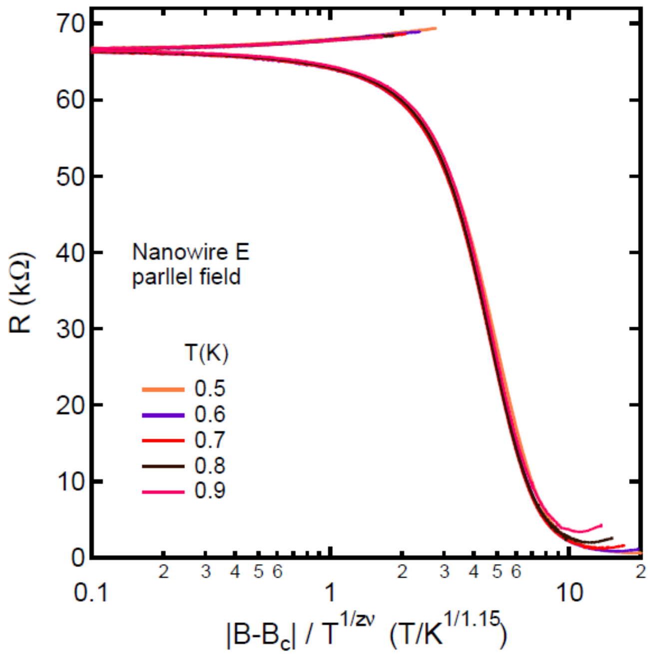


Resistance versus parallel magnetic field for nanowire E. The crossings of the curves indicates two critical fields at lower and higher temperature ranges.

2d scaling analysis for 1d nanowires: data collapse

$$R(B,T) = R_c \Phi_R \left(\frac{|B - B_c|}{T^{1/z\nu}} \right)$$

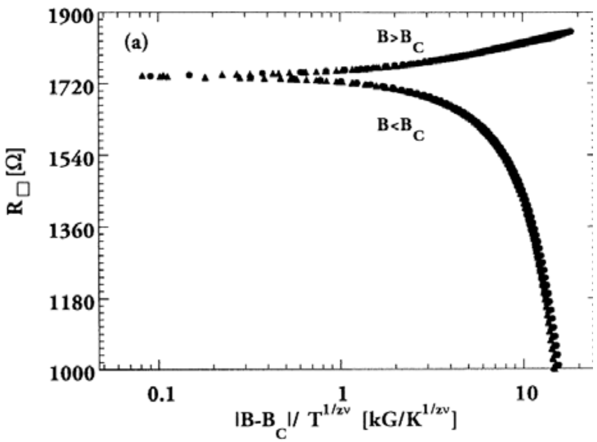




The scaling behavior is accidental and, in fact, misleading, yet it was observed in our all measurements. Our results suggest that the conclusions taken form phenomenological application of the finite-size scaling analysis alone have to be taken with reservation. It is also quite likely that at least in some cases the scaling collapse observed in the past in condensed matter systems was accidental.

We need to revisit magnetic-field-driven QPT in superconducting films (I think)

a-MoGe films, Kapitulnik group

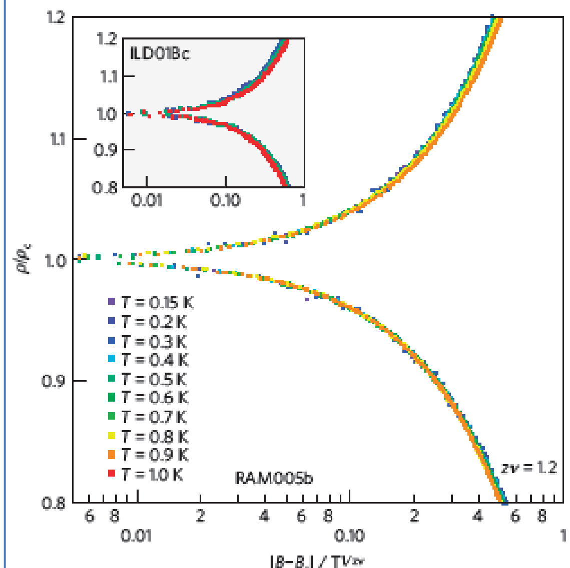


$T = 80 - 110$ mK

$z\nu = 1.3$

Exponents – 4/3 Classical 2D percolation theory

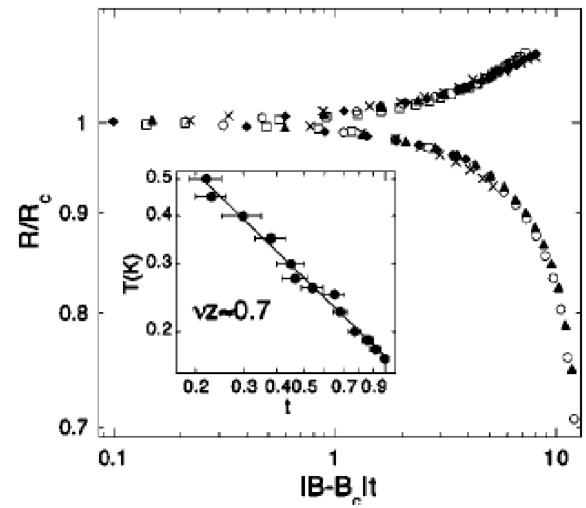
a-InO films, Shahar group



$z\nu = 1.2$

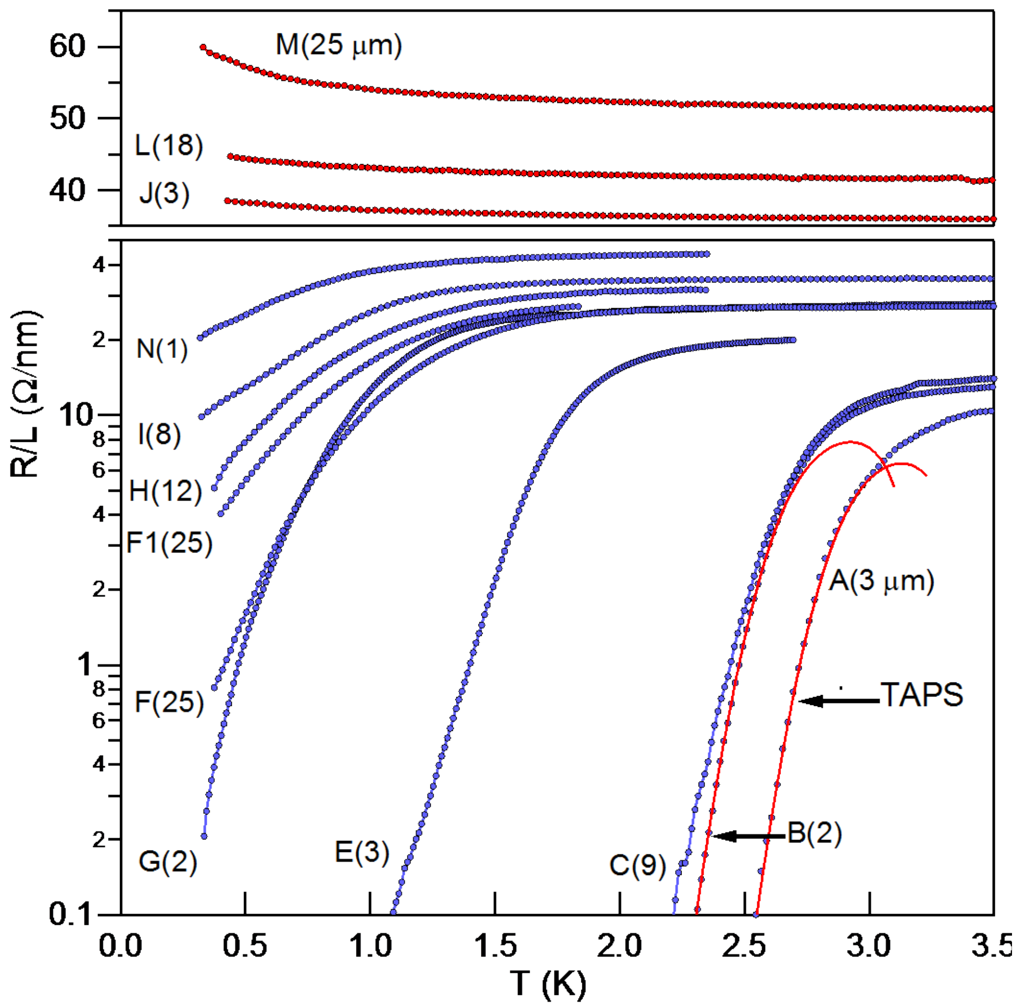
$T = 150 - 1000$ mK

a-Bi, Goldman group



T=150-350 mK

Superconductor – Insulator Transition in 1D Nanowires in Zero Magnetic Field



Amorphous $Mo_{78}Ge_{22}$

Mean free path 0.3 nm

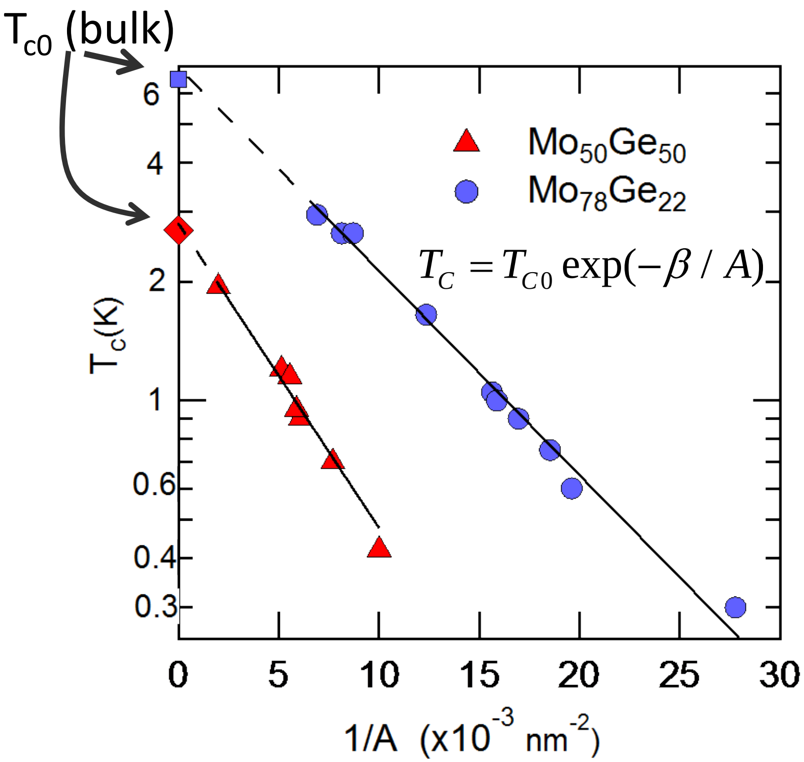
$\rho = 160\ \mu\Omega\ \mathrm{cm}$

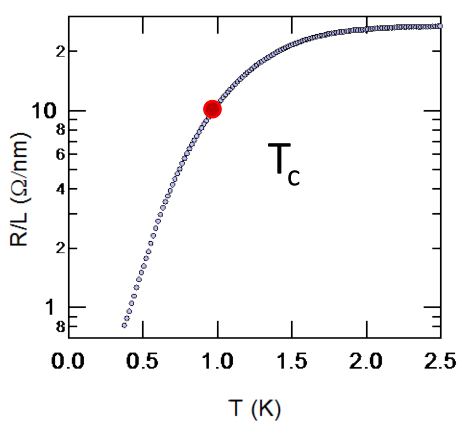
ρ does not depend on thickness down to 1 nm

$$R = \rho \frac{L}{A} \qquad \frac{R}{L} = \rho \frac{1}{A}$$

Superconductivity is controlled by cross sectional area of nanowires.

Variation of mean-filed critical temperature is not understood. Is disorder a pair-breaker?





Suppression of T_c in Superconducting Amorphous Wires

Yuval Oreg[1] and Alexander M. Finkel'stein[2,*] PRL 1999

$$\ln\frac{T_c}{T_{c0}} = -t\Sigma_2(T_{c0}) + t^2\left(\Sigma_3(T_{c0}) + T_{c0}\frac{\partial\Sigma_2(T)}{\partial T}\bigg|_{T=T_{c0}}\Sigma_2(T_{c0})\right) + \ldots$$

$$\Sigma_2(T) = \sum_{n,m=0}^{M}\frac{\Lambda(\epsilon_n + \epsilon_m)}{(n + 1/2)(m + 1/2)}.$$

$$\Lambda(\omega_n) = u\frac{4\pi D}{La}\sum_{q_L,q_a}\frac{1}{Dq_L^2 + Dq_a^2 + \omega_n}$$

$$t = \frac{e^2}{2\pi^2\hbar}R_\square \qquad \omega_n = 2\pi T\left(m + \frac{1}{2}\right)$$

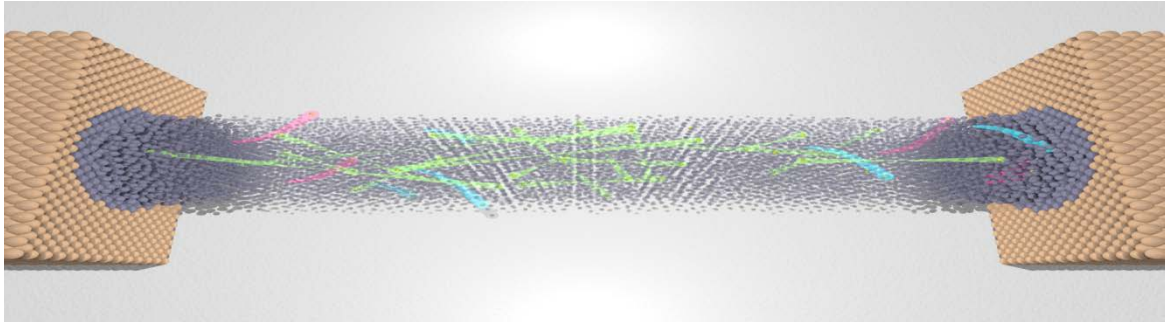
$M = (2\pi T\tau)^{-1}$ determines the upper limit of summation (matrix rank) for $\Lambda(\omega_n, \omega_m)$

Numerical solution using parameters for MoGe nanowires:

There is a 100-fold disagreement with the theory.

The theory predicts much weaker suppression of the critical temperature.

Summary



- Pair-breaking QPT in nanowires indeed takes place and is very well described by the critical pair-breaking theory
- Reliability of the finite-size scaling analysis is questionable
- Magnetic-field driven QPT in superconducting films needs to be revisited
  (pair-breaking critical theory for films is needed !!!; we can easily make strip samples if this helps)
- Suppression of superconductivity in nanowires in zero magnetic field is still a puzzle