Critical field and magneto-resistance of strongly disordered superconductors

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Collaborations, Part 1

B. Sacepe, J. Seidemann & F.Gay, *Neel Institute, Grenoble*
M. Ovadia & K. Michaeli, *Weizmann Institute of Science*
A. Rogachev & K. Davenport, *University of Utah*

ArXiv:1609.07105, *Nature Phys*, 8 Oct. 2018

Collarobations, Part 2

I. Poboiko, Skoltech & Landau Institute

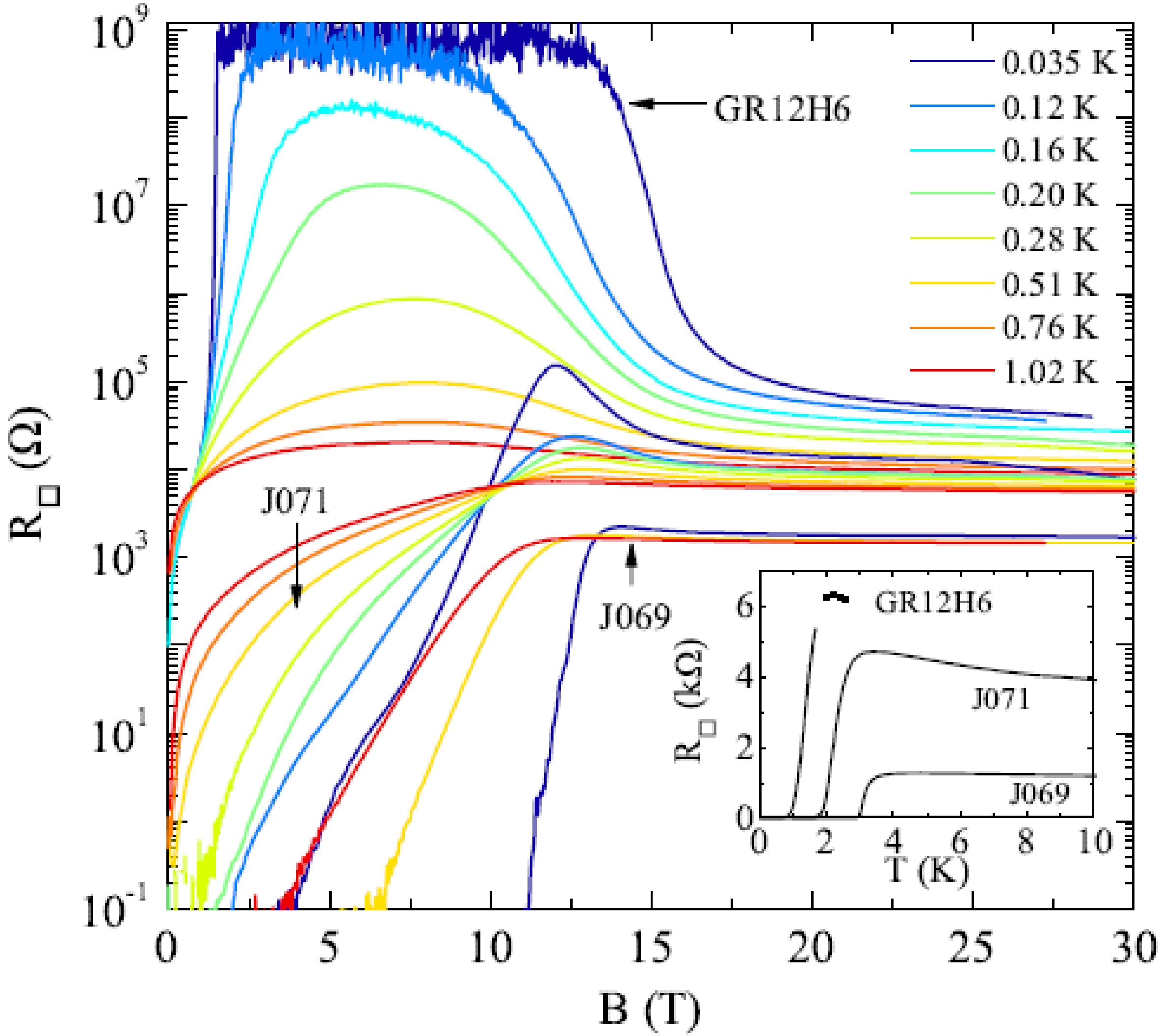
Paraconductivity of pseudogapped superconductors,
Phys. Rev. B 97, 014506 (2018), and to be published

Two weakly related subjects:

1. Moderately disordered superconductors with well-defined $H_{c2}(T)$: theory for low-T anomaly

2. Strongly disordered superconductors near SIT: theory for fluctuation magneto-resistance in weak fields $B \ll H_c(0)$

Evolution of SMT into SIT



B.Sacepe et al, Phys. Rev. B (2015)

Part 1: theory for low-T anomaly of $H_{c2}(T)$

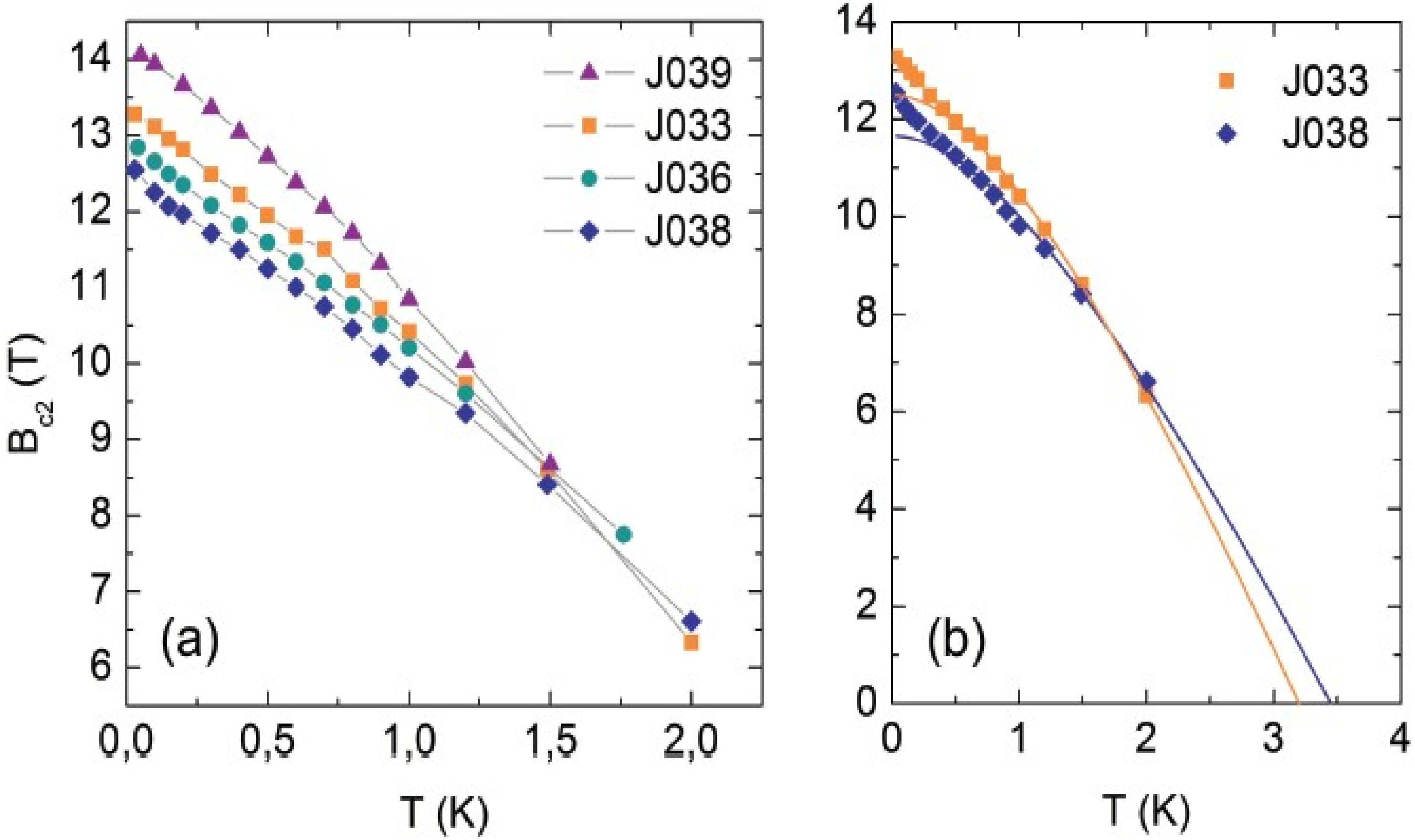


FIGURE 4.9.: (a) B_{c2} versus T curves at low temperature for four samples with different disorder, (b) two of the four samples, with the BCS-fit as solid line.

there is no saturation when zero temperature

is approached, as predicted in the theory for conventional superconductors.

How can one understand a nonzero slope dH_{c2}/dT at T=0 ?

V. M. Galitski and A. I. Larkin, Phys. Rev. Lett. 87, 087001 (2001)

B. Spivak and F. Zhou, Phys. Rev. Lett. 74, 2800 (1995).

F. Zhou and B. Spivak, Phys. Rev. Lett. 80, 5647 (1998).

Explanations in terms of mesoscopic fluctuations

for the “upturn” of the $H_{c2}(T)$ curve at low T

Previous experiments of this kind:

S. Okuma et al., J. Phys. Soc. Jpn. 52, 3269 (1983);

A. F. Hebard and M. A. Paalanen, Phys. Rev. B 30, 4063 (1984).

A. Nodrstrom, U. Dahlborg, O. Rapp, Phys. Rev. B48, 12866 (1993)

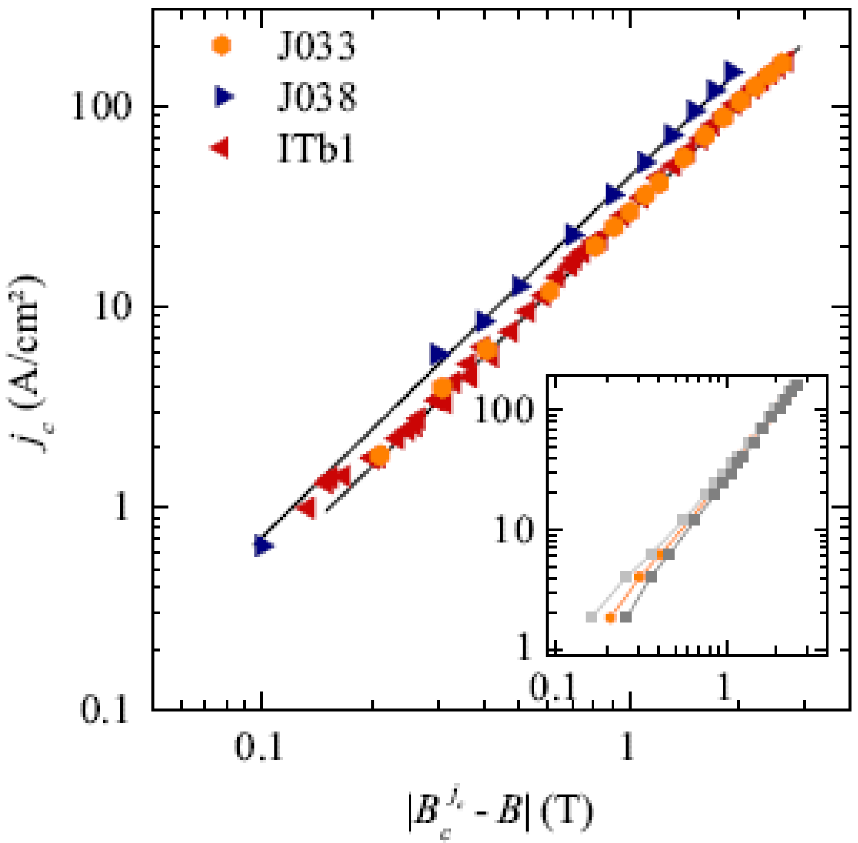


FIG. 3: **Scaling of the critical current density wit magnetic field.** j_c versus $|B_c^{j_c} - B|$. The $B_c^{j_c}$ values ar adjusted to obtain straight lines that are emphasized by blac solid-lines. Inset: the dark grey and light grey curves are th data of sample J033 plotted with $B_c^{j_c} \pm \delta$, where $\delta = 0.05$ T

“Mean-field theory” value:

$$J_c \propto |B_{c2}(0) - B|^{\upsilon}$$

$$\upsilon = 3/2.$$

the measurements lead to a power of 1.62 and 1.65 in sample J033 and J038

How can one reconcile these data ?

1) Finite slope of $H_{c2}(T)$ at T close to zero

2) $J_c \sim (H_{c2} - B)^{3/2}$ at $T = 0$

General idea: the observed effects are due to **combination** of 3D quantum (T=0) phase transition treated within MFA and finite-temperature fluctuation corrections

Interpretation of experimental results within mean-field theory

$$F = \alpha|\Delta(\boldsymbol{r})|^2 + \beta|\Delta(\boldsymbol{r})|^4 + \gamma\left|\left(-i\boldsymbol{\nabla} - \frac{2e}{\hbar c}\boldsymbol{A}(\boldsymbol{r})\right)\Delta(\boldsymbol{r})\right|^2$$

$$\alpha = \nu\left[\ln\frac{T}{T_{c0}} + \psi\left(\frac{1}{2} + \frac{eDB}{2\pi cT}\right) - \psi\left(\frac{1}{2}\right)\right].$$

defines the GL transition line

Magnetic field B is considered to have pure deparing effect:
no vortices for a moment.

At $T \to 0$, $\alpha \approx |1 - B/B_{c2}|$

$$\rho_s(B) = \frac{12}{\pi}\rho_{s0}\left(1 - \frac{B}{B_{c2}(0)}\right)$$

Superfluid density near QPT

$$j_c^{GL}(B) = \frac{4c\,e\,\rho_s(B)}{3\sqrt{3}\hbar\,\xi(B)} = j_c^{GL}(0)\left(1 - \frac{B}{B_c}\right)^{3/2}$$

Where are vortices ?
They are strongly pinned !

Why pinning is so strong ?

Fractal superconductivity near localization threshold

M.V. Feigel'man [a,b], L.B. Ioffe [a,c,d,*], V.E. Kravtsov [a,e], E. Cuevas [f]

Annals of Physics 325 (2010) 1390–1478

Real-space order parameter:

$$\Delta(\xi) = \frac{\lambda}{2}\int d\zeta \eta(\zeta) M(\xi-\zeta)\Delta(\zeta)$$

$$\tilde{\Delta}(\mathbf{r}) = \frac{g}{2}\sum_k \Delta_k \eta_k \psi_k^2(\mathbf{r}).$$

$$\Delta_k = \Delta(\xi_k) \text{ – smooth function}$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1}\tanh(\xi_i/2T).$$

Strong spatial fluctuations

$$\overline{\left(\tilde{\Delta}(\mathbf{r})\right)^2} \equiv \frac{1}{\mathcal{V}}\int d^d\mathbf{r}\tilde{\Delta}^2(\mathbf{r}) = \lambda\int_0^\infty d\xi\eta(\xi)\Delta_c^2(\xi)$$

$$\overline{\tilde{\Delta}(\mathbf{r})} \equiv \frac{1}{\mathcal{V}}\int d^d\mathbf{r}\,\tilde{\Delta}(\mathbf{r}) = \lambda\int_0^\infty d\xi\eta(\xi)\Delta_c(\xi)$$

$$\frac{\left(\overline{\tilde{\Delta}(\mathbf{r})}\right)^2}{\overline{\left(\tilde{\Delta}(\mathbf{r})\right)^2}} = \left(\frac{T_c}{E_0}\right)^\gamma \ll 1$$

Instead of $\Delta(\mathbf{r})$ = const in standard theory

Two sources of de-pairing:

Magnetic field **B** and transport current **j** both produce pair-breaking effects which sum up together:

$$\Gamma_{\mathrm{tot}} = \Gamma_B + \Gamma_j$$

$$\Gamma_B = \frac{2eDB}{c} \qquad \Gamma_j = \hbar D(\nabla\phi)^2 \qquad \nabla\phi = \left(\partial - i\frac{2e}{\hbar c}\mathbf{A}\right)\phi$$

At T=0 critical degree of de-pairing is $\Gamma_c = \Delta_0 = 1.76 T_{c0}$

Electric current is

$$\mathbf{j} = e\nu D(\Gamma_c - \Gamma)\nabla\phi = \frac{2e^2}{c}\nu D^2 B_c \left[\left(1 - \frac{B}{B_c}\right) - \xi_0^2(\nabla\phi)^2\right]\nabla\phi$$

$$\boxed{j_{\mathrm{max}} \propto (B_c - B)^{3/2}} \qquad \xi_0^2 = \frac{\Phi_0}{2\pi B_c}$$

$H_{c2}(T)$ line: role of fluctuations

MFA: $B_{c2}(0) - B_{c2}(T) \sim T^2$ from $\alpha = \nu \left[\ln \frac{T}{T_{c0}} + \psi\left(\frac{1}{2} + \frac{eDB}{2\pi c T}\right) - \psi\left(\frac{1}{2}\right) \right].$

Thermal fluctuations should be accounted for, to estimate corrections to $\rho_s(B)$

$F[\Phi] = \rho_s \left[\nabla \Phi(r) - \frac{2e}{\hbar c} A(r) \right]^2 / 2$ in Gaussian approximation

Nonlinear terms: $u(r) = 2\pi R(r)/a_0$ is the dimensionless displacement field.

$$\delta F = \rho_s a_0^{-1} \hat{z} \cdot [\nabla \varphi(r) \times u(r)] - C\rho_s (\nabla \varphi(r))^2 u^2(r)$$

T -dependent correction to ρ_s comes from the 2^{nd} term

$$\delta \rho_s^{x,y} = -\frac{C\rho_s}{a_0^3} T \sum_n \int dr \langle u(r, \omega_n) \cdot u(0, -\omega_n) \rangle.$$

Dissipative displacement mode **u**(r,t)

$$\eta\partial_t \boldsymbol{u}(\boldsymbol{r},t) + \kappa(\boldsymbol{u}(\boldsymbol{r},t) - \boldsymbol{u}_0(\boldsymbol{r})) = \boldsymbol{f}(t) \equiv \frac{ha_0}{2e}\boldsymbol{j}\times\hat{z},$$

$$\eta = (h/2e)^2\sigma_n/2\pi = \frac{\pi\hbar^2}{2e^2}\sigma_n$$

$$\kappa = \pi\rho_s.$$

$$\boldsymbol{u}(\boldsymbol{r}) - \boldsymbol{u}_0(\boldsymbol{r}) = ha_0(\boldsymbol{j}\times\boldsymbol{n})/2e\kappa$$

$$\delta\boldsymbol{A} = -\hbar\Phi_0\boldsymbol{j}/4e\kappa a_0$$

Matsubara Green's function of $\boldsymbol{u}$

$$G(\boldsymbol{r},\omega_n\neq 0) = a_0^2\delta(\boldsymbol{r})[\eta|\omega_n| + \kappa]^{-1}.$$

Thermal fluctuation correction:

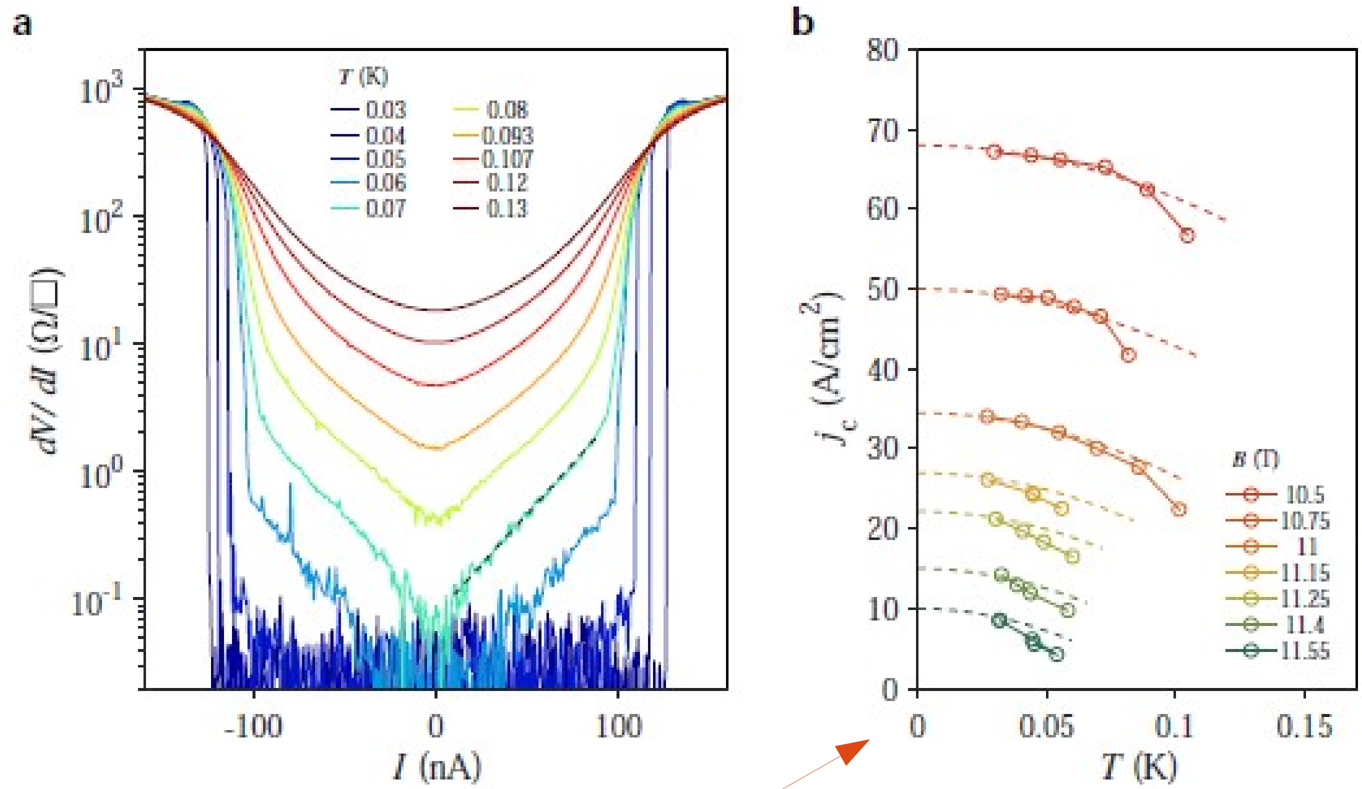
$$\delta\rho_s(T,B) = -\left[T\sum_n[\eta|\omega_n| + \kappa]^{-1} - \int\frac{d\omega}{2\pi}[\eta|\omega| + \kappa]^{-1}\right] = -C\frac{\hbar\sigma_n}{e^2}\frac{T^2}{3\pi\rho_s(B)a_0}.$$

Recall:

$$\rho_s(B) \propto 1 - B/B_{c2}(0)$$

$T_c(B) \sim \rho_S(B) \sim B_{c2}(0) - B$

Experimental $j_c(B,T)$ dependence



$B = 11.25\,\mathrm{T}$

$B_{c2}(0) = 12\,\mathrm{T}$

$$\delta j_c^{GL}(T,B) \propto \frac{\delta\rho_s(T,B)}{\xi_{GL}} \propto \frac{T^2}{\sqrt{B_{c2}(0)-B}}$$

as long as $\delta\rho_s(B,T) \ll \rho_s(B)$

Explanation for the low-T anomaly

1. Criterion for the disappearance of SC state: $\delta\rho_s(T_c, B) = \epsilon\rho_s(0, B)$

(similar to the Lindemann criterion for melting of bulk crystal)

of the order 1

2. Thin films with d << a_0(B) : generalized BKT transition

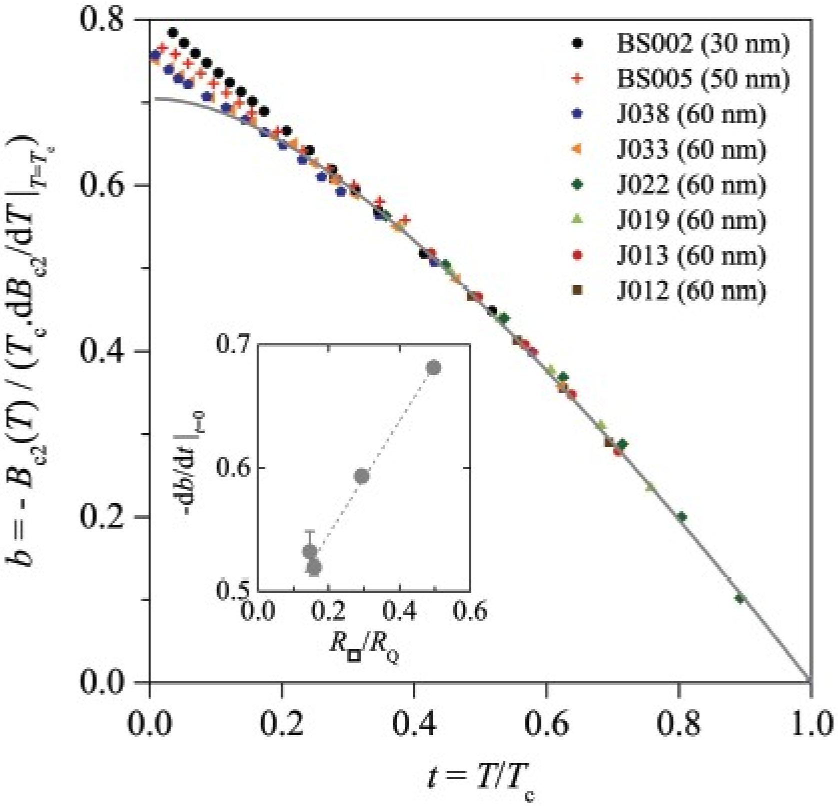
$\rho_s(B, T_{\rm BKT}) = \frac{\chi}{d} T_{\rm BKT}(B)$ with χ, approx 0.5-0.7

A.Yazdani et al, PRL 2013

General interpolation formula:

$$1 - \frac{B_{c2}(T)}{B_{c2}(0)} = \left[1 + \sqrt{1 + C_1^2 \frac{d^2}{\epsilon a_0^2 \chi^2}}\right] \frac{\pi \chi T}{24 \rho_{s0} d}.$$

Comparison with the data at various d



$$1 - \frac{B_{c2}(T)}{B_{c2}(0)} = \left[1 + \sqrt{1 + C_1^2 \frac{d^2}{\epsilon a_0^2 \chi^2}}\right] \frac{\pi \chi T}{24 \rho_{s0} d}$$

$$-\frac{db}{dt} = \begin{cases} K R_\square / R_Q & d \ll a_0 \\ \tilde{g}_0 \sqrt{\frac{1}{\sigma_n R_Q a_0}} + \tilde{K} \frac{R_\square}{R_Q} & d \gg a_0 \end{cases}$$

$\tilde{g}_0$ and $\tilde{K} \approx 0.1$

Inset: Slope $-db/dt$ at zero temperature versus $R_\square / R_Q$.

Experiment: $-db/dt = 0.4 R_\square / R_Q + 0.44$

Coefficients are larger by ~ 5

However, an U(1) – breaking order parameter seems to be of *glassy* nature in our problem.

What about "gauge glass" theory ?

PHYSICAL REVIEW B **68**, 104427 (2003)

Absence of phase stiffness in the quantum rotor phase glass

Philip Phillips   Denis Dalidovich

$$H=-E_C\sum_i\left(\frac{\partial}{\partial\theta_i}\right)^2-\sum_{\langle i,j\rangle}J_{ij}\cos(\theta_i-\theta_j-A_{ij}),$$

The major result: $\rho_s = 0$ and conductivity is finite:

$$\sigma_{\mathrm{bos}}(\omega=0,T\to 0)=\frac{4}{3}\frac{e^2\eta q_0}{hm^4}$$

It is not clear what is the origin and magnitude of η

Evidently, $\rho_s = 0$ does not agree with experimental data

It also contradicts previous theoretical results on XY or Gauge glass at $T > 0$

Classical XY or gauge glass

H. Sompolinsky, G. Kotliar, and A. Zippelius, Phys. Rev. Lett. 5 392 (1984).

Phys. Rev. B 35, 311 (1987).

$\rho_s \sim \underline{q_{EA}\Delta_q} \sim (T_c - T)^3$ (replica method)

System of Josephson junctions as a model of a spin glass

V. M. Vinokur, L. B. Ioffe, A. I. Larkin, and M. V. Feĭgel'man

Zh. Eksp. Teor. Fiz. **93**, 343–365 (July 1987)

The same result by means of *dynamic slow cooling* approach

Theory of Diamagnetism in Granular Superconductors

M. V. Feigelman,[1] and L. B. Ioffe[1,2] Phys. Rev. Lett. **74**, 3447 (1995)

$$\mathbf{j} = -\rho_s^g \delta\mathbf{A}, \quad \rho_s^g = \frac{4\pi^2 c\xi_0^2 n T_g}{\Phi_0^2}\tau^3 \quad \mathbf{j}(\mathbf{a}) = -\mathbf{e_a}(2\pi c\xi_0 n T_g/\Phi_0)\tau^4 Y(a/\tau)$$

Within continuous Parisi RSB scheme $\underline{\rho_s \sim (T_c - T)^3}$

QUANTUM GLASS TRANSITION IN A PERIODIC LONG-RANGE JOSEPHSON ARRAY

D. M. Kagan[1*], *L. B. Ioffe*[1,2], *M. V. Feigel'man*[1] *ЖЭТФ, 1999,* v.***116***, p.*1450*

$$\mathcal{H} = \mathcal{H}_J + \mathcal{H}_C = -E_J \sum_{m,n} \cos\left(\phi_n - \phi_m - \frac{2e}{\hbar c}\int \mathbf{A}\cdot d\mathbf{l}\right) + \frac{(2e)^2}{2}\sum_{m,n} \hat{C}^{-1}_{m,n} \frac{\partial}{\partial \phi_m}\frac{\partial}{\partial \phi_n}$$

$$s_m = e^{i\phi_m} \qquad G_{m,n}(\tau) = -\langle T_\tau s_m(\tau) s_n^\dagger(0)\rangle$$

Vanishes at QPT

$$\mathcal{G}(\tilde{t}) = \frac{1}{\sqrt{\tilde{t}}} f\left(\frac{\tilde{t}}{\tau_1}\right) + \mathcal{G}_1 \exp\left(-\frac{\tilde{t}}{\tau_0}\right) \qquad \tau_0 = \frac{\sqrt{32/27}}{\tilde{g}^3}\frac{1}{b^2} \qquad \mathcal{G}_1 = \sqrt{\frac{27}{2}}\tilde{g}^3\, b.$$

This solution is more similar to 1-step RSB

Results for diamagnetic response

$$\chi_{\mathcal{M}}(\omega) \propto \sqrt{i\omega}\ln\omega. \qquad \omega \gg (J/J_c - 1)^2 \alpha^{-3/2}$$

$$\chi_{\mathcal{M}}(\omega) \propto \frac{J_c^3}{(J_c - J)^3}\omega^2, \qquad \omega \ll \frac{C_l (J - J_c)^2}{e^2 \alpha^{5/2}}.$$

In disordered phase

Scaling in glassy phase: $\rho_s \propto (J - J_c)$

Conclusions, part I

1) Anomaly of nonzero dH_{c2}/dT slope at $T=0$ is due to

a) linear dependence of $\rho_s \sim (H_{c2} - B)$ at $T=0$ which is a feature of a 3D quantum glass transition

b) very strong pinning of vortices in SC with high disorder

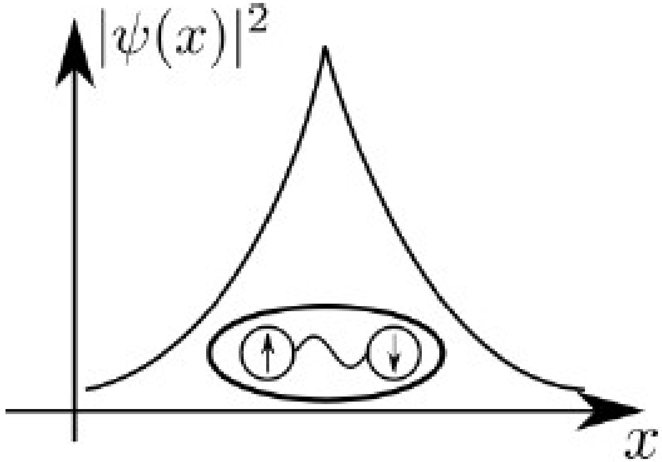
c) dissipative fluctuations of some bosonic mode (vortex fluctuations) at $T>0$

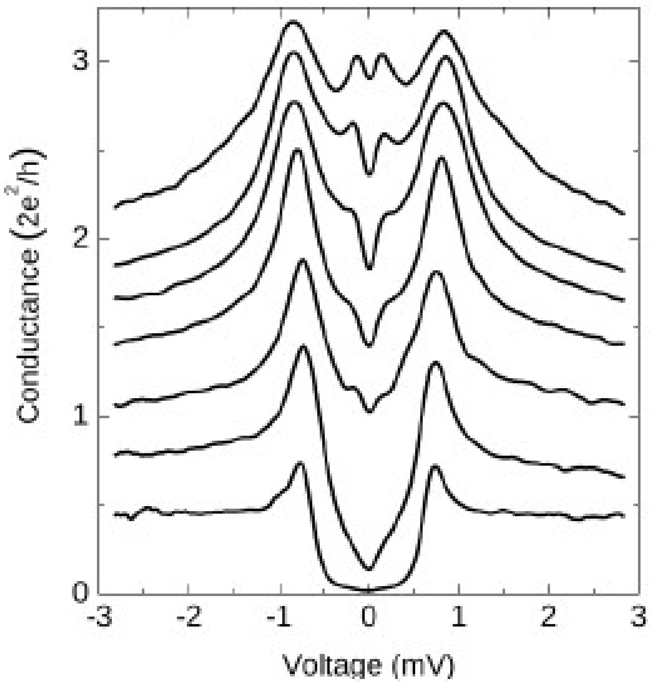
2) For the same SC material, the slope dH_{c2}/dT grows linearly with $1/d$

3) Theory of phase stiffness near quantum glass transition is to be developed

Part 2

Superconductors with a pseudogap
near transition temperature:
Aslamazov-Larkin paraconductivity
as a tool to determine low-temperature
coherence length





- Amorphous thin films of InO_x: strongly disordered superconductor (close to SIT)
- Cooper attraction between electrons leads to well-developed pseudogap $\Delta_P > \Delta$

B. Sacépé, T. Dubouchet, C. Chapelier, M. Sanquer, M. Ovadia, D. Shahar, M. Feigel'man, and L. Ioffe, Nature Physics **7**, 239 (2011)

T. Dubouchet et al., arXiv:1806.00323v1 (2018)

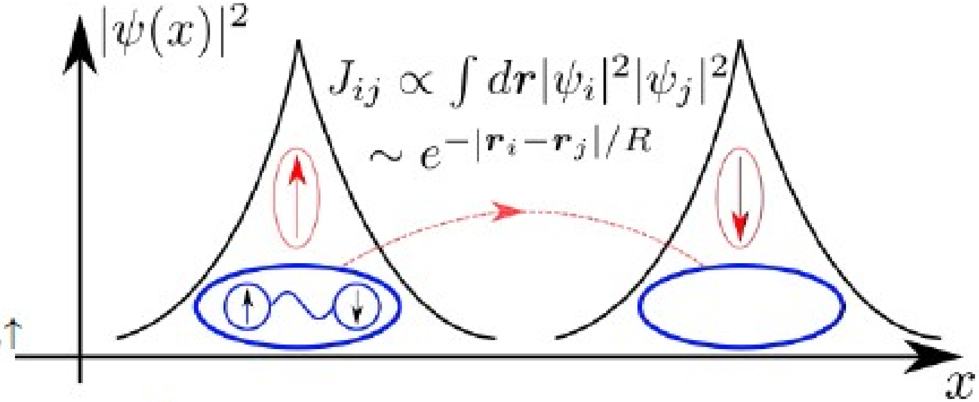
Talk by Claude Chapelier tomorrow morning

Theoretical model

Anderson pseudospins:

$$S_i^z = \frac{1}{2}(1-\hat{a}^\dagger_{i,\uparrow}\hat{a}_{i,\uparrow}-\hat{a}^\dagger_{i,\downarrow}\hat{a}_{i,\downarrow})$$

$$S_i^+ = \hat{a}^\dagger_{i,\uparrow}\hat{a}^\dagger_{i,\downarrow}, \quad S_i^- = \hat{a}_{i,\downarrow}\hat{a}_{i,\uparrow}$$



$$H = -2\sum_i \varepsilon_i S_i^z - \frac{1}{2}\sum_{ij} J_{ij}(S_i^+S_j^- + h.c.)$$

Anderson localized band:

$$\nu(\varepsilon) = \nu_0\theta(W-|\varepsilon|), \quad \nu_0 = \frac{1}{2W}$$

M. V. Feigel'man et al., Ann. Phys. 325, 1390 (2010)

Mean-field approximation

$$H = -2\sum_i \varepsilon_i S_i^z - \frac{1}{2}\sum_{ij} J_{ij}(S_i^+ S_j^- + h.c)$$

- Local order parameter:

$$\Delta_i = \sum_j J_{ij}\left\langle S_j^- \right\rangle, \quad S_i^- = \hat{a}_{i,\downarrow}\hat{a}_{i,\uparrow}$$

- Mean-field effective Hamiltonian:

$$H_{MF} = -2\boldsymbol{h}_i \hat{\boldsymbol{S}}_i, \quad \boldsymbol{h}_i = (\mathrm{Re}\,\Delta_i, \mathrm{Im}\,\Delta_i, \varepsilon_i)$$

- Self-consistency equation (BCS-like!):

$$1 = \frac{J}{2}\int d\varepsilon \nu(\varepsilon) \frac{\tanh\left(\beta\sqrt{\varepsilon^2 + |\Delta|^2}\right)}{\sqrt{\varepsilon^2 + |\Delta|^2}}$$

$$\boxed{T_c = \frac{4e^\gamma}{\pi} W e^{-1/g}, \quad \Delta(0) = 2We^{-1/g} \quad g = \nu_0 J}$$

Relevant parameters

- BCS-like parameters:
  - Coupling constant: $g = \nu_0 J = J/2W \ll 1$
  - Transition temperature: $T_c \sim W \exp(-1/g)$
  - "Zero-temperature" coherence length: $\xi_0 = R/\sqrt{g}$
- Temperature is close to T_c (normal phase):

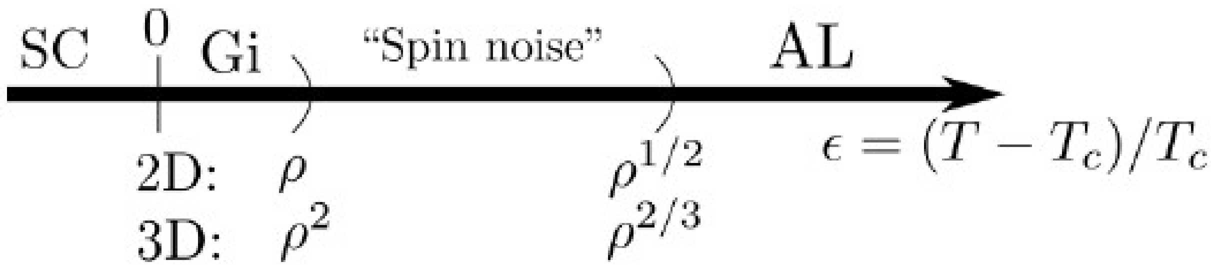
$$\epsilon = \ln \frac{T}{T_c} \approx \frac{T - T_c}{T_c} \ll 1$$

- "Mean-field" parameter: inverse effective number of neighbors:

$$\rho = K^{-1} = (\nu_0 T_c \xi_0^d)^{-1} \sim \frac{g^{d/2} e^{1/g}}{R^d} \ll 1$$

The key unknown parameter: $\xi_0 = R/\sqrt{g}$

H_{c2} ($T \ll T_c$) cannot be defined experimentally due to giant R(B) peak



- Gaussian region: Aslamazov-Larkin result (NB: twice larger!):

$$\sigma_0(T) = \frac{e^2}{\hbar} \times \begin{cases} 1/8\epsilon, & (2D) \\ 1/16\xi_0\sqrt{\epsilon}, & (3D) \end{cases}$$

- “Spin noise” region: non-universal AL:

$$\sigma(T) \sim C(T)\sigma_0(T), \quad C(T) \sim 1$$

- “Ginzburg” region: strong fluctuations.

1. AL contribution is **the only** fluctuation term in pseudo-gapped superconductor.
2. It can be larger than normal-state conductivity since “normal state” is insulating

Gaussian approximation: “TDGL”

- The Dyson equation for order parameter propagator:

$$\left\langle \hat{L} \right\rangle_{\varepsilon}^{-1} = \hat{J}^{-1} - \langle \Pi_i \rangle_{\varepsilon} ,$$

$$L(\omega, \boldsymbol{q}) = i \left\langle \Delta \bar{\Delta} \right\rangle_{\omega, \boldsymbol{q}} = \frac{1/2\nu_0}{i\omega\tau - \epsilon - q^2 \xi_0^2},$$

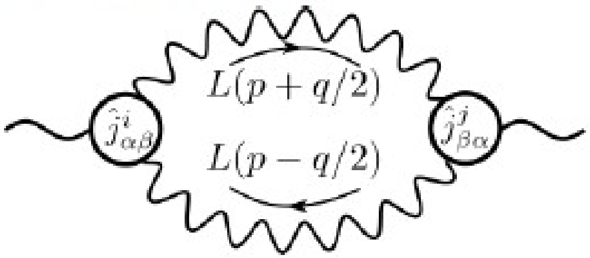
$$\boxed{\epsilon = \ln \frac{T}{T_c} \ll 1, \quad \xi_0 = R/\sqrt{g}, \quad \tau = \pi/4T}$$

- NB: in conventional superconductors one has:

$$\tau_{\mathrm{metal}} = \pi/8T$$

Gaussian approximation: conductivity

- Current-current correlation function:



$$Q_{ij}(\boldsymbol{r},t) = -i\left\langle j_i(\boldsymbol{r},t) j_j(0,0)\right\rangle^R .$$

- Conductivity (Kubo):

$$\sigma = i\frac{\partial Q(\boldsymbol{q}=0,\omega)}{\partial\omega}$$

- Twice the Aslamazov-Larkin (due to $T\tau$):

$$\sigma_{AL} = \frac{e^2}{\xi_0^{d-2}\epsilon^{2-d/2}}\frac{8}{d}T\tau\int\frac{(d\boldsymbol{P})P^2}{(1+P^2)^3} = \frac{e^2}{\hbar}\times\begin{cases} 1/8\epsilon, & (2D) \\ 1/16\xi_0\sqrt{\epsilon}, & (3D) \end{cases}$$

Range of applicability of the theory: $\rho^{2/(4-d)} \leq \varepsilon << 1$

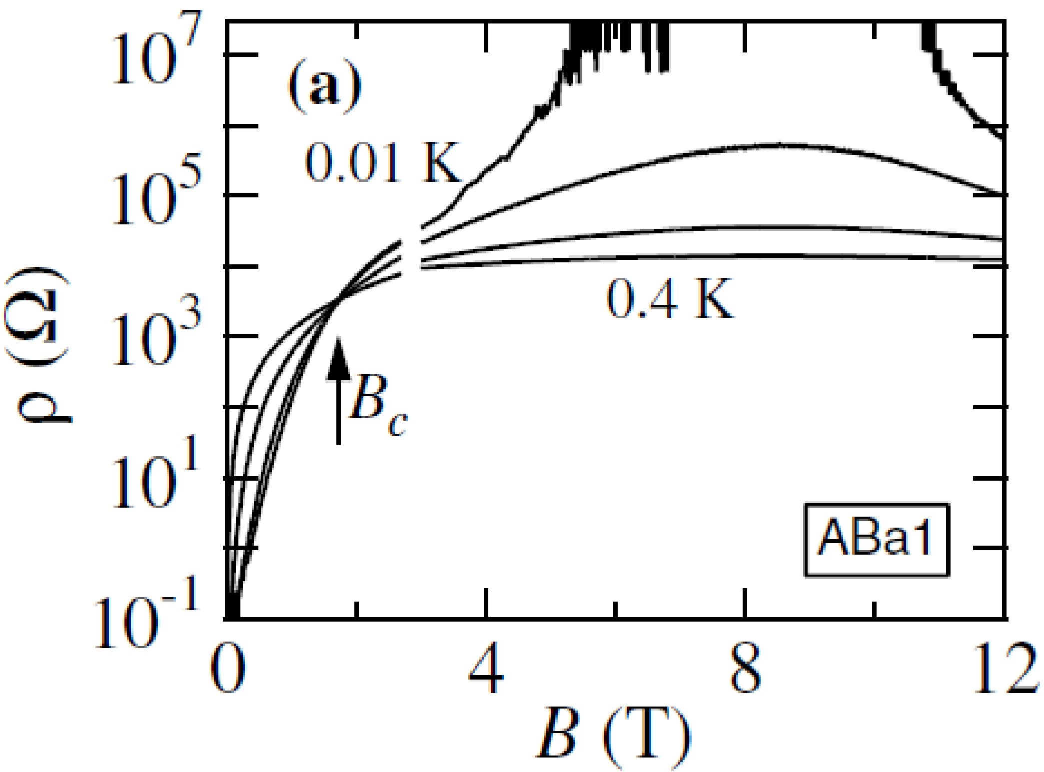
$$\rho = \begin{cases} \dfrac{1}{16\nu_0\xi_0^2 T} & \text{(2D)} \\[2ex] \dfrac{1}{16\sqrt{\pi}\,\nu_0\xi_0^3 T} & \text{(3D).} \end{cases}$$

How do we know the value of ξ_0 ?

Usually we extract it from the low-temperature value of H_{c2}

PRL **94,** 017003 (2005)

G. Sambandamurthy,[1] L. W. Engel,[2] A. Johansson,[1] E. Peled,[1] and D. Shahar[1]



H_{c2} is poorly defined for superconductors close to SIT

Alternative approach: to measure fluctuation contribution to magneto-resistance at relatively weak magnetic fields and close to the critical temperature

In pseudo-gaped superconductors, Aslamazov-Larkin para-conductivity is the only singular contribution, so we can look for its dependence on magnetic field.

EFFECT OF A MAGNETIC FIELD ON FLUCTUATIONS ABOVE T_c†

E. ABRAHAMS, R. E. PRANGE‡ and M. J. STEPHEN

Physica 55, 230 (1971)

They used time-dependent Ginzburg-Landau model

In a presence of magnetic field, order parameter propagator satisfies the following equation:

$$\nu_0 \left(\epsilon - \xi_0^2(\hat{\boldsymbol{p}} - 2e\boldsymbol{A})^2 - i\omega\tau\right) L_\omega(\boldsymbol{r}, \boldsymbol{r}') = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

Landau gauge, $A_y = Hx$;

$$\psi_{n,p_y,p_z}(x,y,z) = e^{ip_y y + ip_z z}\psi_n(x)$$

$$\left[-\xi_0^2\partial_x^2 + (2eH\xi_0)^2\left(x - \frac{p_y}{2eH}\right)^2\right]\psi_n(y) = \lambda_n\psi_n(y)$$

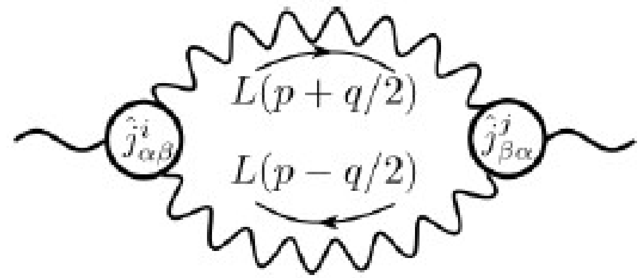
$$a = \frac{1}{\sqrt{m\omega}} = \frac{1}{\sqrt{2eH}}, \quad \omega = 4eH\xi_0^2 = \frac{2\xi_0^2}{a^2} \equiv 2h$$

$$\lambda_n = h(2n+1), \quad \psi_n(x) = \frac{e^{-y^2/2a^2}H_n(x/a)}{\pi^{1/4}a^{1/2}\sqrt{2^n n!}}$$

$$L_\omega(\boldsymbol{r}, \boldsymbol{r}') = \int_0^{L_x/a^2}(dp_y)\int(dp_z)\sum_n \frac{e^{ip_y(y-y')+ip_z(z-z')}\psi_n(x - p_y a^2)\psi_n(x' - p_y a^2)}{\epsilon + h(2n+1) + p_z^2\xi_0^2 - i\omega\tau}$$

Now we use the above result for calculating Aslamazov-Larkin paraconductivity

Current-current correlation function:



$$\Gamma_n \equiv \epsilon + h(2n+1) + p_z^2\xi_0^2 \qquad Q_{ij}(r,t) = -i\left\langle j_i(r,t)j_j(0,0)\right\rangle^R .$$

$$\sigma_{xx} = 8\nu_0^2\xi_0^4e^2T\int\frac{(d\Omega)}{\Omega^2}\cdot\frac{1}{V}\int(dr)(dr')\hat{p}_x\mathrm{Im}L_R(\Omega,r,r')\hat{p}_x'\mathrm{Im}L_R(\Omega,r',r) =$$

$$= \frac{1}{\pi}\left(\frac{4\xi_0^2e}{a}\right)^2T\tau^2\int(d\Omega)\int(dp_z)\sum_{nn'}\frac{(\hat{p}_x)_{nn'}(\hat{p}_x)_{n'n}}{(\Gamma_n^2+\Omega^2\tau^2)(\Gamma_{n'}^2+\Omega^2\tau^2)}$$

Next we integrate over Ω and obtain:

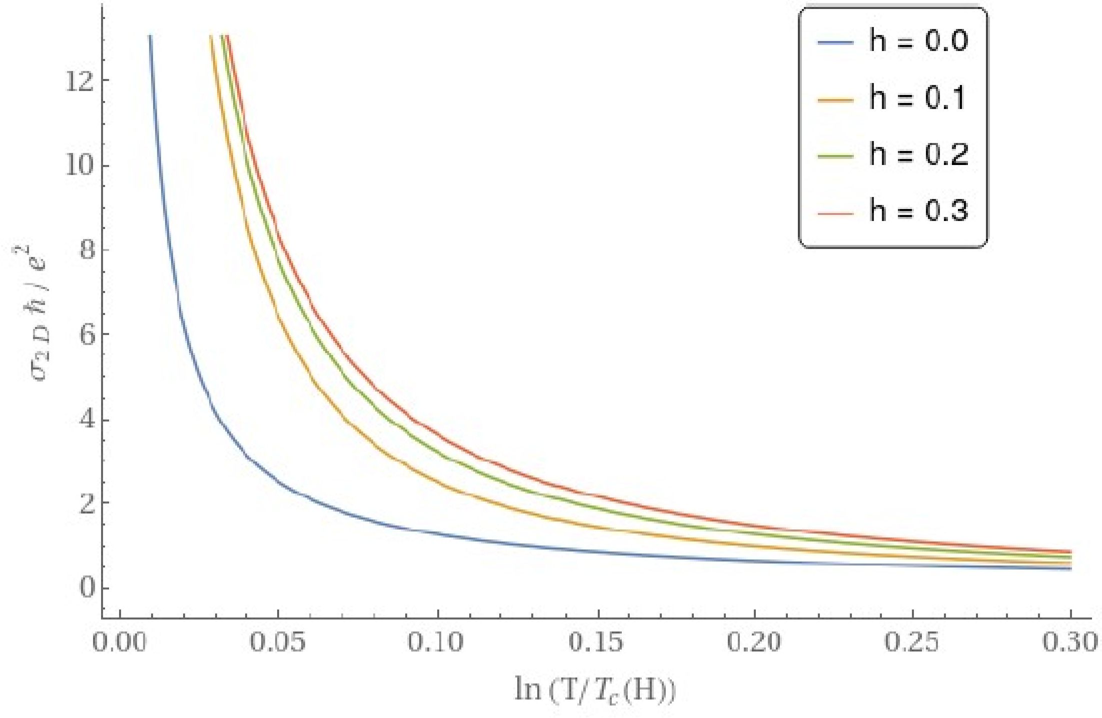
$$\sigma_{xx} = \left(\frac{4\xi_0^2e}{a}\right)^2\frac{T\tau}{2\pi}\int(dp_z)\sum_{nn'}\frac{(\hat{p}_x)_{nn'}(\hat{p}_x)_{n'n}}{\Gamma_n\Gamma_{n'}(\Gamma_n+\Gamma_{n'})} = 2e^2h^2\int(dp_z)\sum_{n=0}^{\infty}\frac{n+1}{\Gamma_n\Gamma_{n+1}(\Gamma_n+\Gamma_{n+1})}$$

Next, we calculate this expression for various thickness d of the film

2D limit $d \le \xi_0$

$\psi(z) = d \ln \Gamma(z)/dz$

$$\frac{\sigma_{xx}(h,\epsilon)}{\sigma_{AL}(\epsilon)} = 16\epsilon h^2 \sum_{n=0}^{\infty} \frac{n+1}{\Gamma_n \Gamma_{n+1}(\Gamma_n + \Gamma_{n+1})} = \boxed{2\left(\frac{\epsilon}{h}\right)^2 \left(\psi\left(\frac{1}{2}+\frac{\epsilon}{2h}\right) - \psi\left(1+\frac{\epsilon}{2h}\right) + \frac{h}{\epsilon}\right)}$$



$h = \xi_0^2 B/\Phi_0$

Quasi-2D case: d >> ξ_0

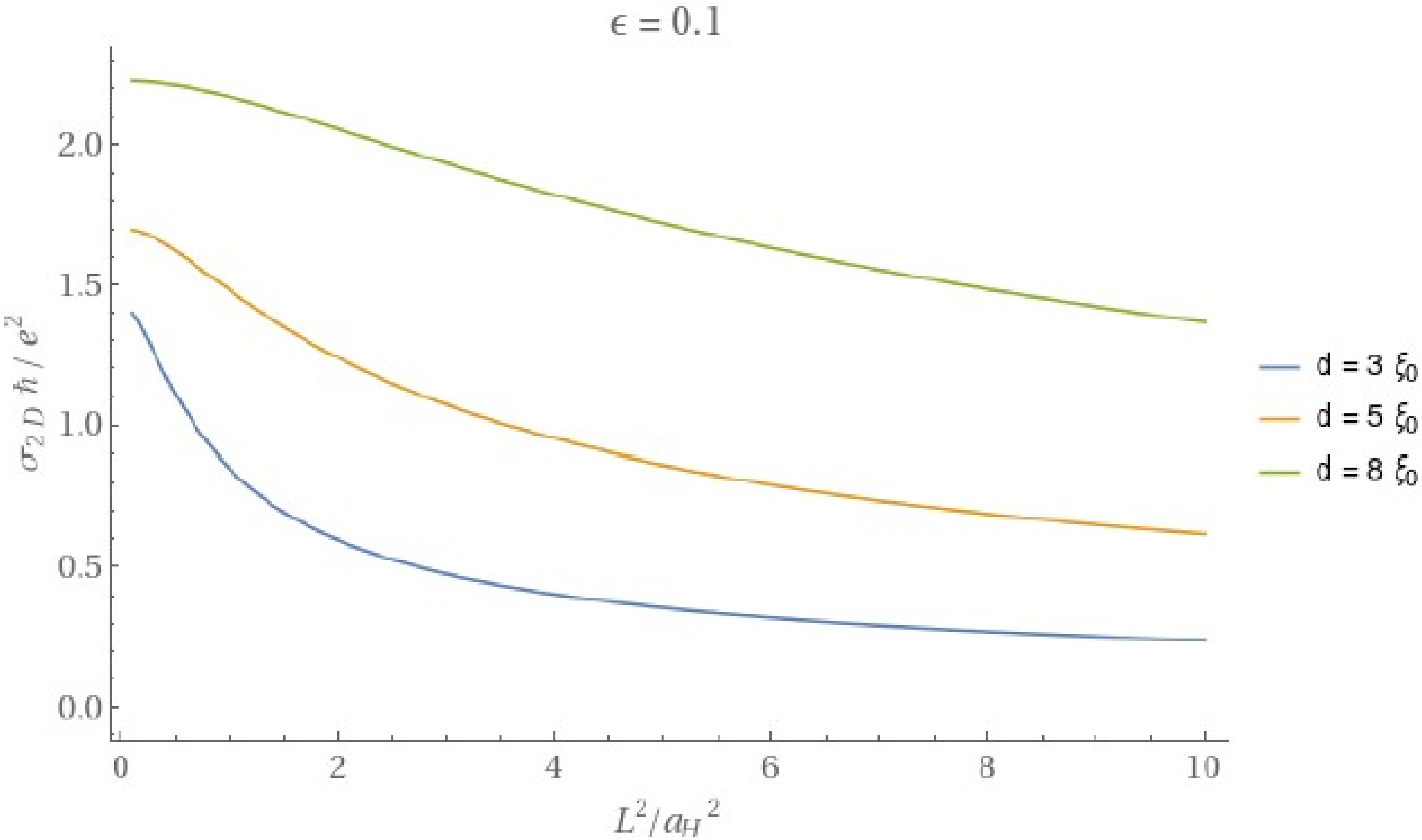
$$\frac{\sigma(h,l,\epsilon)}{\sigma_{AL}(\epsilon)/L} = 16h^2\epsilon \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{n+1}{\Gamma_{n,m}\Gamma_{n+1,m}(\Gamma_{n,m}+\Gamma_{n+1,m})}, \quad \Gamma_{n,m} \equiv \epsilon + h(2n+1) + \pi^2 m^2/l^2$$

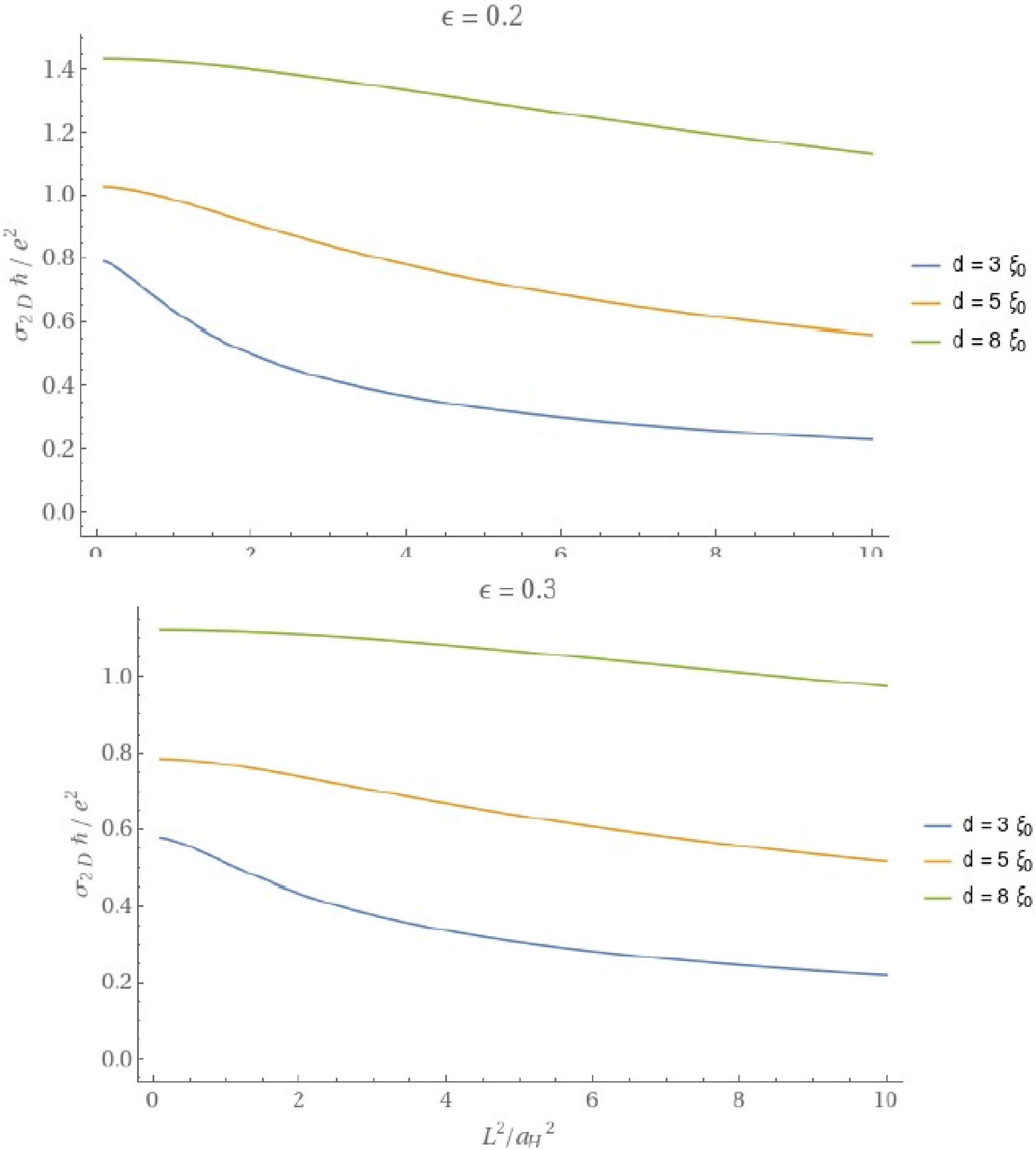
l = d/ξ_0

$$\frac{\sigma_{xx}(h,l,\epsilon)}{\sigma_{AL}(\epsilon)/L} = \frac{2\epsilon}{h} \sum_{m=0}^{\infty} \left(\frac{p_m^2+\epsilon}{h} \left(\psi\left(\frac{p_m^2+\epsilon}{2h}+\frac{1}{2}\right) - \psi\left(\frac{p_m^2+\epsilon}{2h}+1\right)\right)+1\right)$$

Note that this function depends effectively on two dimensionless parameters, ϵ/h and l^2h.

Magneto-resistance for various thickness and reduced temperature ε




$\epsilon = 0.2$
$\sigma_{2D}\,\hbar\,/\,e^2$
d = 3 ξ_0
d = 5 ξ_0
d = 8 ξ_0
$\epsilon = 0.3$
$\sigma_{2D}\,\hbar\,/\,e^2$
d = 3 ξ_0
d = 5 ξ_0
d = 8 ξ_0
$L^2/{a_H}^2$

Conclusions, part 2

1. Aslamazov-Larkin para-conductivity due to fluctuating pairs is the only singular contribution near T_c of pseudo-gaped superconductors.
2. It is found to be twice larger than in usual dirty superconductors.
3. It can be much larger than "normal-state" conductivity
4. Magnetic-field dependent para-conductivity is calculated and can be used to determine low-temperature coherence length ξ_0