

Superinsulators: one color QCD with Cooper pairs

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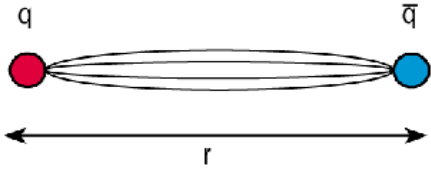
**Nips laboratory, INFN and Department of Physics
and Geology University of Perugia**

Coll:

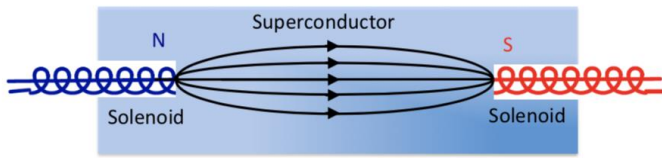
- **Luca Gammaitoni, University of Perugia** [arXiv:1806.00823](https://arxiv.org/abs/1806.00823)
- **Carlo A. Trugenberger, SwissScientific**
- **Valerii Vinokur, Argonne National Laboratory** [arXiv:1807.01984](https://arxiv.org/abs/1807.01984)

P. Sodano, C.A. Trugenberger, MCD, Nucl. Phys. B474 (1996) 641

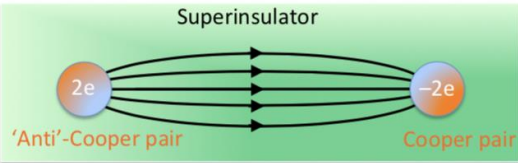
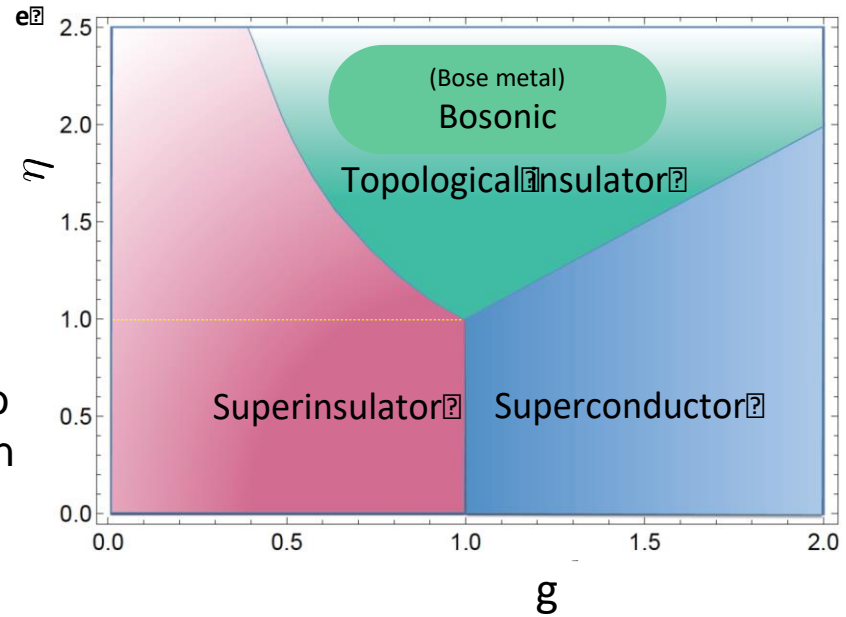
$T=0$



quarks bound by (chromo)-
electric strings in a
condensate of
(chromo)-magnetic
monopoles (Mandelstam,
't Hooft, Polyakov)



mirror analogue to
vortex formation in
type II
superconductors



Polyakov's magnetic
monopole condensation
⇒ **linear confinement**
of Cooper pairs by
electric strings ⇒
one color QCD

Nature of phases: universal
All microscopic details enter
 g , η and the Pearl length of SC and
string tension of SI

Superinsulation: realization and proof of confinement
by monopole condensation and asymptotic freedom
in solid state materials

Cooper pairs



Quarks

Superconductor

$$R = 0$$

$$G = \infty$$



S duality

Mandelstam 'tHooft
Polyakov

Superinsulator

$$R = \infty$$

$$G = 0$$

't Hooft 1978 (Nucl. Phys. B138 (1978) 1) :

“....absolute confinement is realized in a phase which is in many respects similar to the superconducting phase. In a certain sense it is the extreme opposite (“superinsulator”)

- **theoretically predicted in 1996 in the SIT**
(P. Sodano, C.A. Trugenberger, MCD, Nucl. Phys. B474 (1996) 641)
- **experimentally observed in:**
 - In₂O₃ films** (Sambandamurthy et al, Phys.Rev.Lett. 94(2005) 017003)
 - TiN films** (Baturina et al, Nature 452 (2008) 613)
- **confirmed in NbTin films in 2017**
(Vinokur et al, Scientific Reports 2018)

- superinsulating state **dual** to the superconducting state

Superconductor

Cooper pair condensate
and pinned vortices



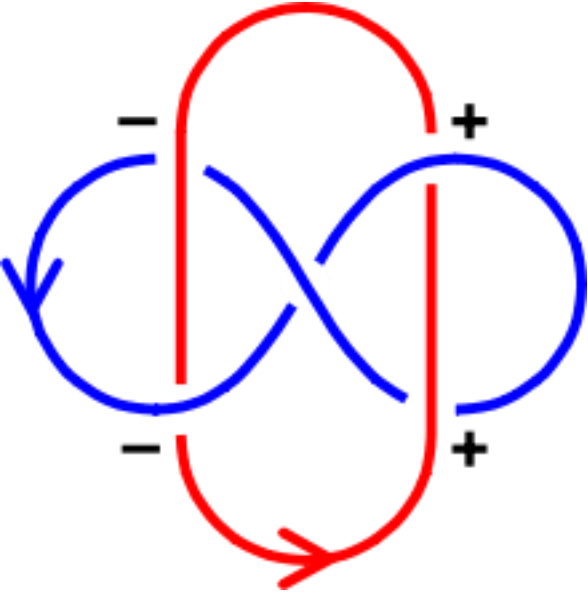
Superinsulator

Vortex condensate and
localized Cooper pairs

- **Cooper pairs** and **vortices** are the relevant degrees of freedom
- SIT is driven by the competition between charge (Cooper pairs) and vortex **degrees of freedom:**
topological interactions, Aharonov-Bohm and Aharonov-Casher **gauge invariance**
- tuning parameters η and g depend on material characteristics

2d

Q_μ M_μ world lines of elementary charges (Cooper pairs) and vortices



$$S_{\text{linking}} = \int d^3x i 2\pi Q_\mu \epsilon_{\mu\alpha\nu} \frac{\partial_\alpha}{-\nabla^2} M_\nu$$

integer linking numbers do not contribute to the partition function; contribution for infinitely extended world-lines of charges and vortices

local formulation:

$$S^{\text{CS}} = \int d^3x \left[i \frac{\kappa}{2\pi} a_\mu \epsilon_{\mu\alpha\nu} \partial_\alpha b_\nu + i \sqrt{\kappa} a_\mu Q_\mu + i \sqrt{\kappa} b_\mu M_\mu \right]$$

two emergent gauge fields a_μ (vector) and b_μ (pseudovector); emergent mixed Chern–Simons term, **U(1) x U(1) symmetry**



need regularization

PT BREAKING TOPOLOGICAL STATES OF MATTER

FQH states:

- gapped in the bulk; gapless edge excitations;
- low energy effective field theory \Rightarrow pure Chern-Simons term:
background independent;
ground state degeneracy on manifolds with non-trivial topology;
quasiparticles have fractional charge and statistics;

• Wen's idea (1992):

$$j_\mu \propto \epsilon_{\mu\nu\alpha} \partial_\nu b_\alpha$$

conserved matter current, b_μ a U(1)
pseudovector gauge field if j_μ is a charge
current

- topological field theory at low energy (Euclidean),

$$S = i \int d^3x \frac{k}{2\pi} b_\mu \epsilon_{\mu\nu\alpha} \partial_\nu b_\alpha$$



CS, P (T) breaking

gauge invariant, one derivative,
dominant at large distances

PT INVARIANT TOPOLOGICAL STATES OF MATTER

can we have P and T symmetries?

- **Sodano, Trugenberger, MCD (1996)**

two fluids model, **2d**

$$\vec{j}_\mu \propto \epsilon_{\mu\nu\alpha} \partial_\nu b_\alpha$$

conserved charge current, b_μ is a **pseudovector**

$$\vec{\phi}_\mu \propto \epsilon_{\mu\nu\alpha} \partial_\nu a_\alpha$$

conserved vortex current a_μ is a **vector**

$$S = i \int d^3x \frac{k}{2\pi} a_\mu \epsilon_{\mu\nu\alpha} \partial_\nu b_\alpha$$



mixed CS \equiv BF, P(T) invariant
U(1) x U(1)

- add kinetic term for the emergent gauge fields

$$S_{\text{TM}} = \int d^3x \frac{1}{4e_f^2} f_{\mu\nu} f_{\mu\nu} + i \frac{k}{2\pi} a_\mu \epsilon_{\mu\nu\alpha} \partial_\nu b_\alpha + \frac{1}{4e_g^2} g_{\mu\nu} g_{\mu\nu}$$

these are the terms that are allowed by gauge invariance with minimum number of derivatives

3d

$$j_\mu \propto \epsilon_{\mu\nu\alpha\beta} \partial_\nu b_{\alpha\beta}$$



charge current, $b_{\mu\nu}$ pseudotensor

$$\phi_{\mu\nu} \propto \epsilon_{\mu\nu\alpha\beta} \partial_\alpha a_\beta$$



vortex current, a_μ vector

$$S_{BF} = i \frac{\kappa}{2\pi} \int d^4x b_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} \partial_\alpha a_\beta$$



BF , P(T) invariant
U(1) x U(1)

+ kinetic for a_μ and $b_{\mu\nu}$

$$S_{TM} = \int d^4x \frac{1}{12e_g^2} h_{\mu\nu\rho} h_{\mu\nu\rho} - \frac{i\kappa}{8\pi} b_{\mu\nu} \epsilon_{\mu\nu\lambda\rho} f_{\lambda\rho} + \frac{1}{4e_f^2} f_{\mu\nu} f_{\mu\nu}$$

BF theory provides a generalization of fractional statistics to arbitrary dimensions (Semenoff, Szabo), (3+1): particles around vortex strings

S_{TM} was first proposed in 2 and 3d as a field theory description of topological phases of condensed matter systems in 1996 (Sodano, Trugenberger, MCD)

$$S_{TM} = \int d^4x \frac{1}{12e_g^2} h_{\mu\nu\rho} h_{\mu\nu\rho} - \frac{i\kappa}{8\pi} b_{\mu\nu} \epsilon_{\mu\nu\lambda\rho} f_{\lambda\rho} + \frac{1}{4e_f^2} f_{\mu\nu} f_{\mu\nu}$$

S_{TM} : generalization of CS mass to BF theory, **topological mass generation**

- a_μ and b_μ ($b_{\mu\nu}$) acquire a topological mass $m = (k e_f e_g) / 2\pi$
 - k is a dimensionless parameter, it determines the ground state degeneracy on manifold with non trivial topology and the statistics
 - $[e_f^2] = m^{-d+3}$ $[e_g^2] = m^{d-1}$ naively irrelevant (first one marginal in 3d) but necessary to correctly define the limit $m \rightarrow \infty$ (pure CS limit)
- (Dunne, Jackiw, Trugenberger, 1990)
- they enter in the phase structure of the theory

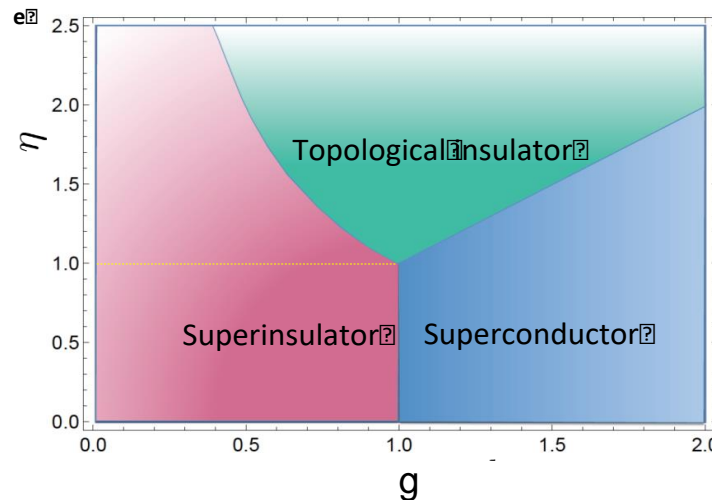
2d

compact gauge theory \Rightarrow lattice regularization, lattice spacing l

$$Z = \sum_{\{M_\mu, Q_\mu\}} \int \mathcal{D}a_\mu \mathcal{D}b_\mu \exp -S$$

$$S = \sum_x \frac{l^3}{4e_f^2} f_{\mu\nu} f_{\mu\nu} + i \frac{l^3}{\pi} a_\mu \epsilon_{\mu\nu\alpha} d_\nu b_\alpha + \frac{l^3}{4e_g^2} g_{\mu\nu} g_{\mu\nu} + \\ + il\sqrt{2}a_\mu Q_\mu + il\sqrt{2}b_\mu M_\mu$$

charge world-lines Q_μ represent the singularities in the dual field strength f_μ
 vortex world-lines M_μ represent singularities in the dual field strength g_μ



$$\eta = \frac{\pi m l G(ml)}{\mu}$$

with $G(ml)$ diagonal part of the lattice kernel:

$$l^2 (m^2 - \nabla^2) G(x - y) = \delta(x - y)$$

$$g = e_f / e_g$$

NON RELATIVISTIC MODEL

$$S^{2D} = \int d^3x \frac{ik}{2\pi} a_\mu \epsilon_{\mu\nu\alpha} \partial_\nu b_\alpha + \frac{1}{2e_f^2 \mu_P} f_0^2 + \frac{\epsilon_P}{2e_f^2} f_i^2 + \frac{1}{2e_g^2 \mu_P} g_0^2 + \frac{\epsilon_P}{2e_g^2} g_i^2$$
$$+ i\sqrt{k} a_\mu Q_\mu + i\sqrt{k} b_\mu M_\mu$$

e_g^2 charge energy

ϵ_P dielectric permittivity

e_f^2 vortex energy

μ_P magnetic permittivity

$g \equiv e_f/e_g$ tuning parameter

$g \Leftrightarrow 1/g$ duality

light velocity in the medium

$$v_c = \frac{1}{\sqrt{\epsilon_P \mu_P}}$$

$$m^{CS} = \mu_P \frac{e_f e_g}{\pi}$$

$$e^2_g = e^2/d$$

$$e^2_f = \pi^2/(e^2 \lambda_{\perp})$$

$K = \lambda_{\perp} / \xi$ Landau
parameter

$\lambda_{\perp} = \lambda_L^2/d$ Pearl
length $2d$

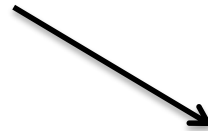
$\lambda_L =$ London
penetration depth

$$\alpha = e^2 / (\hbar c)$$

$d =$ film thickness

$\approx \xi$ coherence length

$$\eta = (1/\alpha) \zeta(K, v_c)$$



quantum behavior

characteristic of the material

SUPERINSULATING PHASE

induced effective action $S^{\text{eff}}(A_\mu)$ for the electromagnetic gauge potential A_μ

2d : M_μ condense, Q_μ diluted

$$\exp(-S^{\text{eff}}) = \sum_{M_\mu} \exp \sum_x [-\gamma (M_\mu - e l^2 F_\mu / \pi)^2]$$

$$\gamma = g\mu\eta$$

Villain approximation of compact QED in 2d

$$S_{\text{QED}} = \gamma/2\pi^2 \sum_x [1 - \cos(2e l^2 F_\mu)]$$

(Polyakov)

M_μ can be open ending in magnetic monopole (instanton) $d_\mu M_\mu = m$

m  tunneling event

dense phase \Rightarrow monopole gas

$$S_{\text{Top}} = \sum_x \frac{2\pi^2}{l^3 \gamma} m \frac{1}{-\nabla^2} m$$

Wilson loop: its expectation value measures the potential between static external test charges q ($2e$) and anti-charges

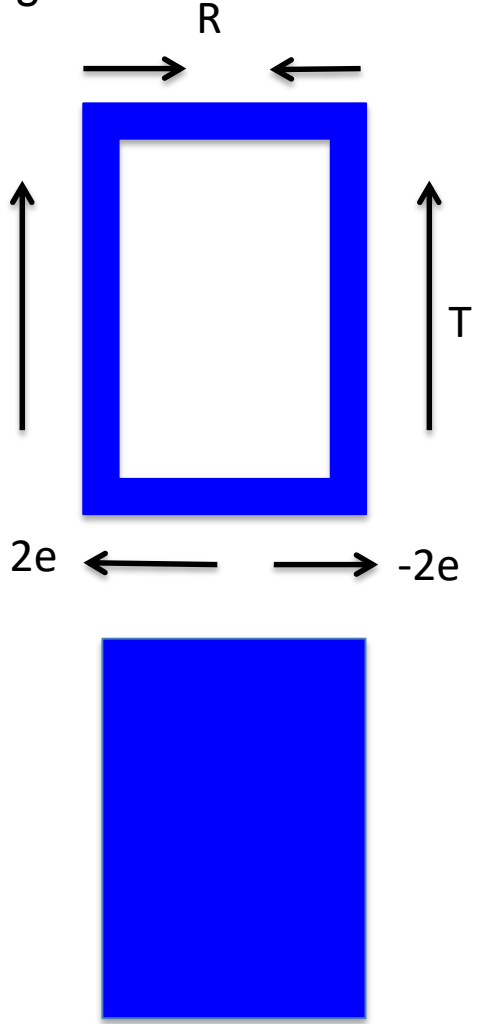
$$W(C) \equiv \exp i \int_C q \sum_{\mu} q_{\mu} A_{\mu}$$

rectangular loop, for $T \rightarrow \infty$

$$\langle W(C) \rangle \propto_{T \rightarrow \infty} \exp -V(R) T$$

$$\langle W(C) \rangle \propto \exp -\sigma A \Rightarrow V(R) = \sigma R$$

A = area enclosed by the loop C
 σ emergent scale, string tension



dual Meissner effect, charge confinement in a monopole condensate, **true also in 3d**
 \Rightarrow superinsulation can exist also in 3d

2d

$$\langle W(C) \rangle \equiv \langle \exp i \int q \sum_x q_\mu A_\mu \rangle$$

expansion in power of $q \Rightarrow$

$$\Rightarrow \langle F_\mu(x) F_\nu(y) \rangle_0 \propto \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\nabla^2} \right) \delta_{x,y}$$

without monopoles

$$\langle F_\mu(x) F_\nu(y) \rangle \propto \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\nabla^2 - m_\gamma^2} \right) \delta_{x,y}$$

with monopoles

F_μ dual field strength of the e.m. field

(Polyakov)

instantons disorder the system \Rightarrow short range correlations

photon acquires a dynamical mass m_γ

$$m_\gamma = \frac{1}{l} 4\pi^{\frac{3}{2}} \gamma \exp(-\pi\gamma)$$

$$\langle W(C) \rangle = \int \mathcal{D}B_{\mu\nu} \exp \left(-S(B_{\mu\nu}) + i \int_{\text{surface}} B_{\mu\nu} d\sigma_{\mu\nu} \right)$$



$B_{\mu\nu} \neq b_{\mu\nu}$

$\sigma_{\mu\nu}$ parametrize the surface enclosed by the loop C

integration over $B_{\mu\nu}$ leads to an induced action for $\sigma_{\mu\nu}$

confining string action: (Quevedo and Trugenberger, Polyakov)

$$\langle W(C) \rangle = \exp - S_{\text{conf. string}}(\sigma_{\mu\nu})$$

first term in the derivative expansions of $S_{\text{conf. string}}(\sigma_{\mu\nu}) \propto \sigma A$

$\sigma = \text{string tension}$

string tension (Polyakov; Kogan and Kovner; Quevedo, Trugenberg, and MCD)

2d:

$$\sigma = \frac{\pi^{3/2}}{l^2} \exp\left(-\frac{\gamma}{16\pi}\right)$$

3d:

$$\sigma = \frac{1}{64\pi l^2} K_0(\gamma/2)$$

confining string picture like QCD :

linear confinement of Cooper pairs into neutral "U(1) mesons"

typical size

$$d_{\text{string}} \simeq ((v_c)/\sigma)^{1/2}$$

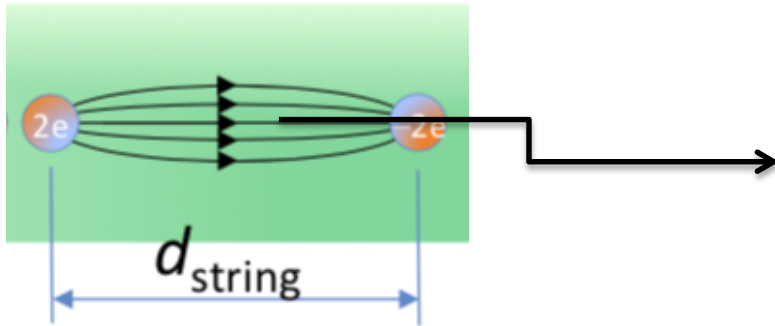
2d:

$$d_{\text{string}} \simeq l \exp\left(K \frac{g\eta c}{v_c}\right) \quad K \text{ numerical constant}$$

near SIT :

$$g \approx 1/\eta \quad v_c = 1/\sqrt{\mu_P \epsilon_P} \ll c$$

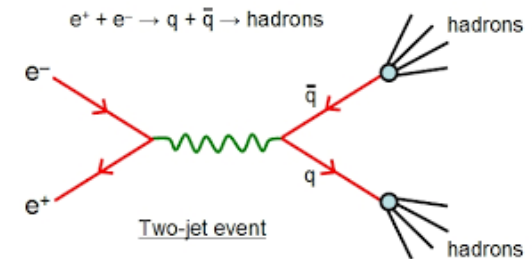
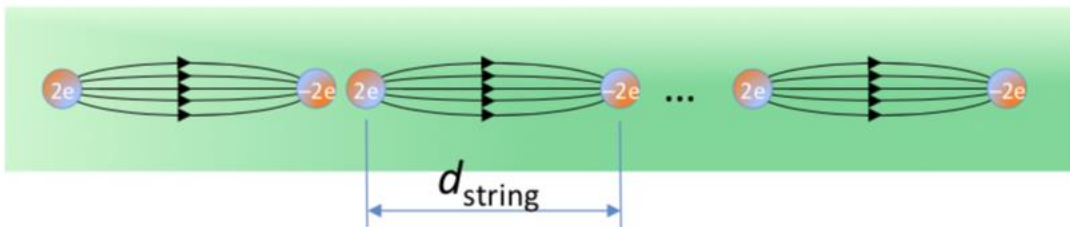
$$d_{\text{string}} \gg l$$



$$W_{\text{string}} = 1/m_{\gamma}$$

(Caselle, Panero and VDACCHINO, JEHP 02 (2016) 180)

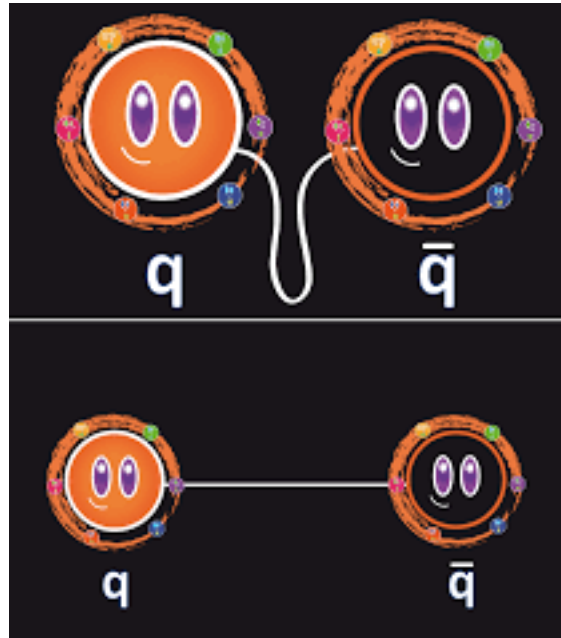
long strings unstable, string fragmentation via creation of charge-anticharge pairs like formation of hadron jets at LHC \Rightarrow creation of neutral mesons ($V \approx 2m_{\text{CP}}$, V applied voltage)



dynamical deconfinement transition ($T=0$): increase V to $V_t \approx \sigma L$, L size of the system \Rightarrow breaking of neutral mesons:

- creation of strips of 'normal' insulator where current can flow
- large current fluctuations
- large current fluctuations recently observed in InO_x films (Tamir et al, [arXiv:1806.09492](https://arxiv.org/abs/1806.09492))

HINT OF ASYMPTOTIC FREEDOM



reverse of the confinement on scales smaller than the typical string size

SIT: string scale can be inferred from experimental data

$$d_{\text{string}} = \hbar v_c / K T_{\text{CBKT}} \quad (v_c \text{ speed of light in the material})$$

$K T_{\text{CBKT}}$ energy required to break up the string

d_{string} scale associated with this energy

TiN films: $T_{\text{CBKT}} = 60 \text{ mK}^\circ$

$$v_c = 5 \times 10^5 \text{ ms}^{-1} \quad (\text{Vinokour et, Nature 452 (2008) 613; Ann.Phys. 331 (2013) 236})$$

$$d_{\text{string}} \leq 60 \text{ } \mu\text{m}$$

study of formation of superinsulators in TiN films of different sizes: samples of size $\leq 20 \mu\text{m}$ **metallic behaviour**

(Kalok et al, arXiv:1004.5153)

DECONFINEMENT CRITICALITY

critical behavior of confining string at finite temperature:

$\sigma(T)$ = string tension (same thing as mass for a particle) vanishes at a critical temperature $T_C \Rightarrow$ **deconfinement:**

infinitely long strings on the microscopic scale and Cooper pairs at the end are liberated

correlation length $\xi \propto 1/\sqrt{\sigma}$

$$\sigma \propto \exp\left(\frac{-\text{const}}{\sqrt{|T/T_C-1|}}\right), \quad R \propto \exp\left(\frac{\text{const}}{\sqrt{|T/T_C-1|}}\right), \quad 2d$$

(Yaffe and Svetitski) \Rightarrow **Berezinski-Kosterlitz-Thouless critical scaling**

$$\sigma \propto \exp\left(\frac{-\text{const}}{|T/T_C-1|}\right), \quad R \propto \exp\left(\frac{\text{const}}{|T/T_C-1|}\right), \quad 3d$$

InO_x films: “more” 3d than TiN and NbTiN films ($d \gg \xi \equiv$ superconducting coherence length) (Ovadia et al, Scientific Reports 5 (2015) 13503) **Vogel-Fulcher-Tamman criticality \equiv behaviour of confining strings in 3d** (Gammaitoni, Trugenberger, Vinokour, MCD)

THANK YOU!!!!