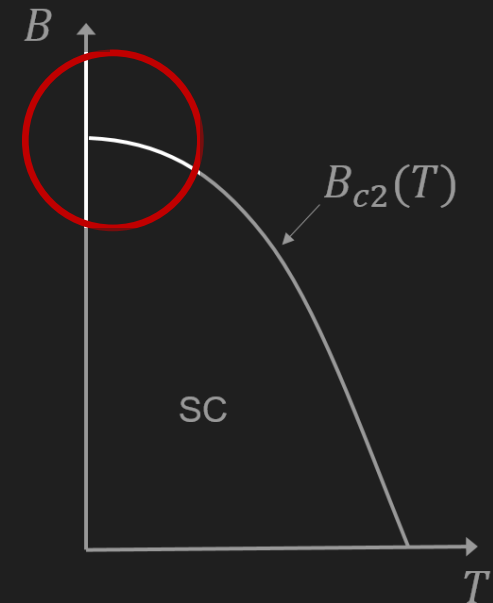


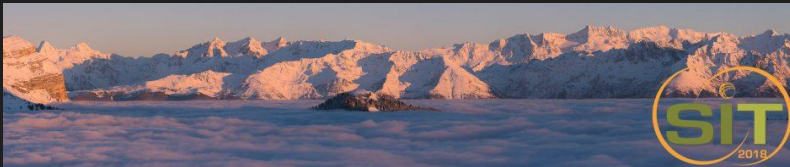
Quantum meets classical phase transition: Low-T anomaly in disordered superconductors near B_{c2}

Benjamin Sacépé

Néel Institute, CNRS & Univ. Grenoble Alpes



Villard de Lans, October 8-12, 2018



Experiments



Johanna Seidemann
Néel Institute



Fédéric Gay
Néel Institute



Maoz Ovadia
Weizmann Institute

MoGe samples



Kevin Davenport
Univ. of Utah



Andrey Rogachev
Univ. of Utah

Theory



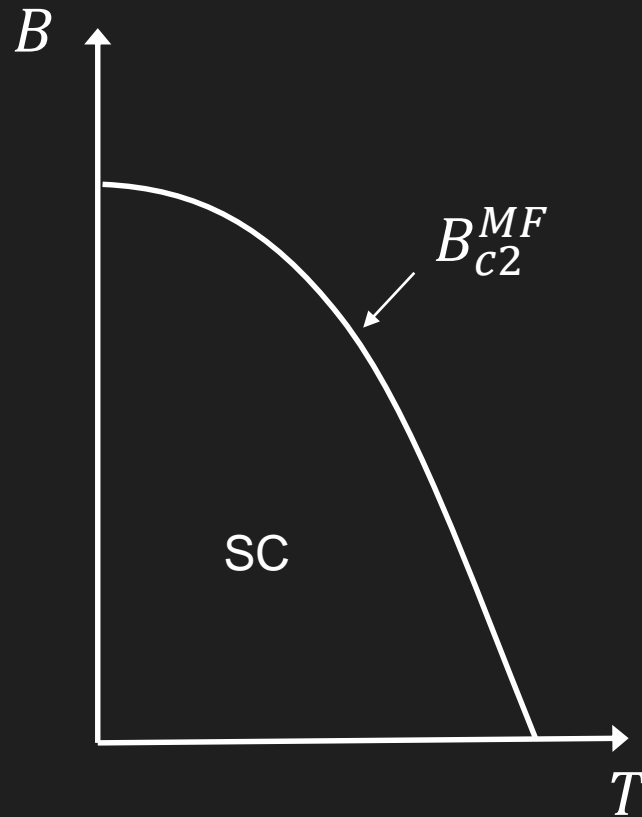
Karen Michaeli
Weizmann Institute



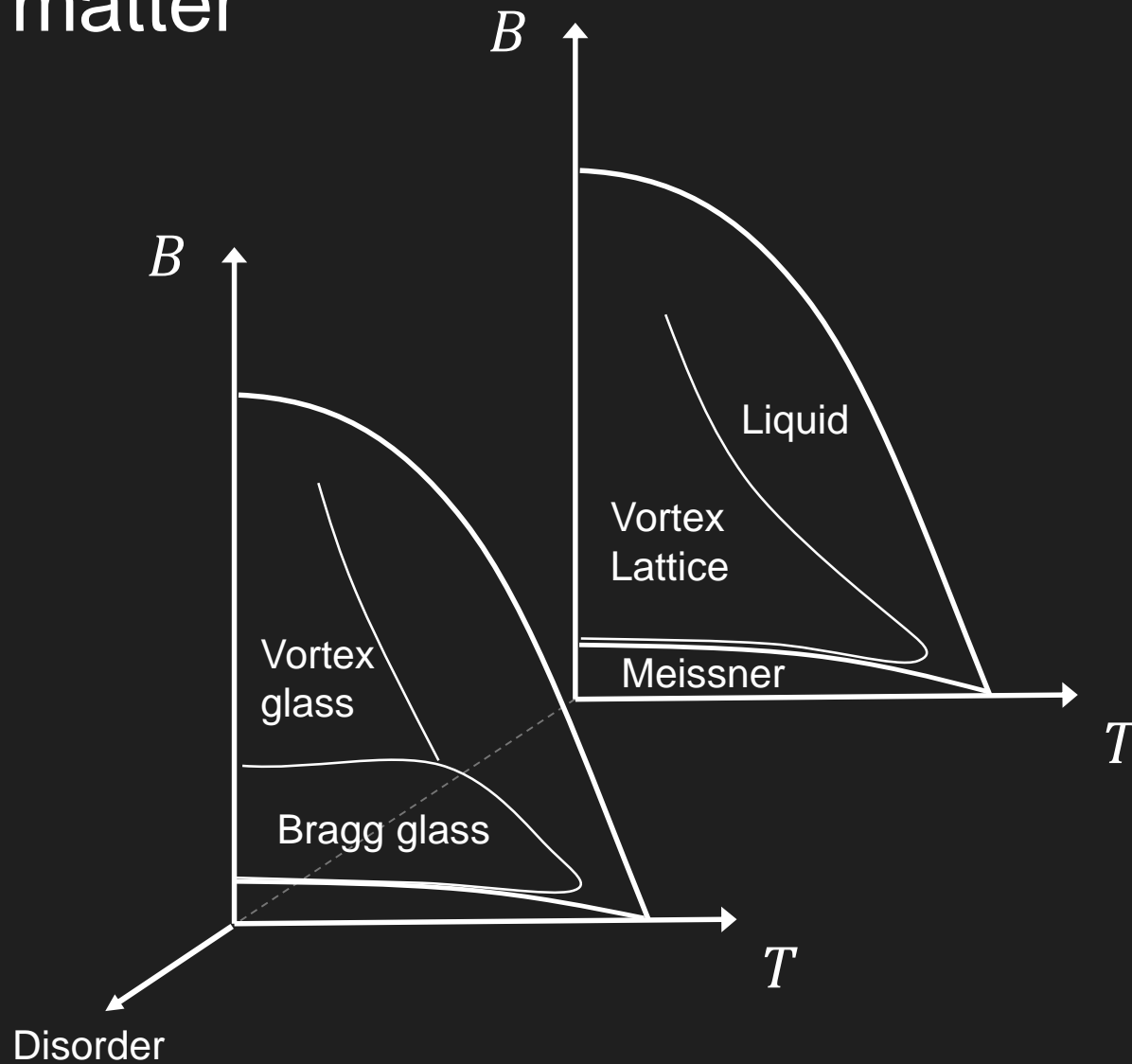
Mikhail Feigel'man
(And the cat)
Landau Institute

+ Many fruitful discussions with Shahar's group

B - T phase diagram

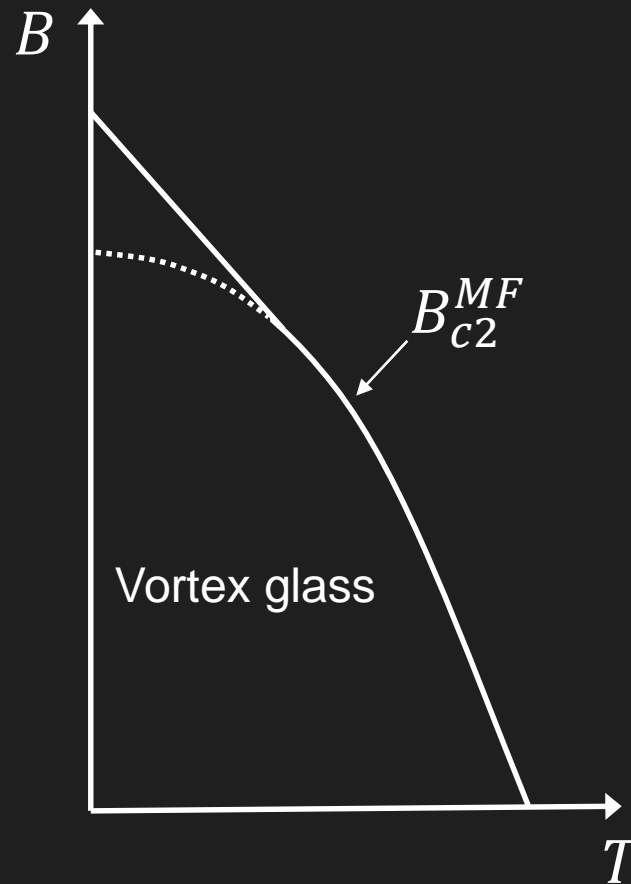


Vortex matter



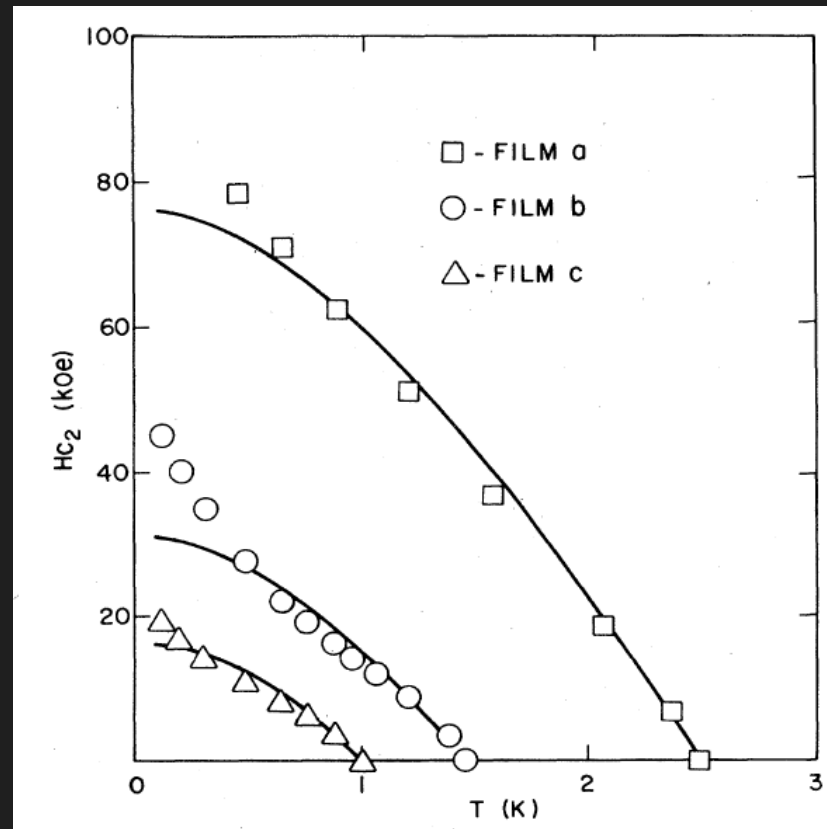
Fisher, Fisher, Huse PRB ('91)
Blatter, et al. RMP ('94)
Kwok, et al. Rep. Prog. Phys. ('16)

(Very) dirty superconductors

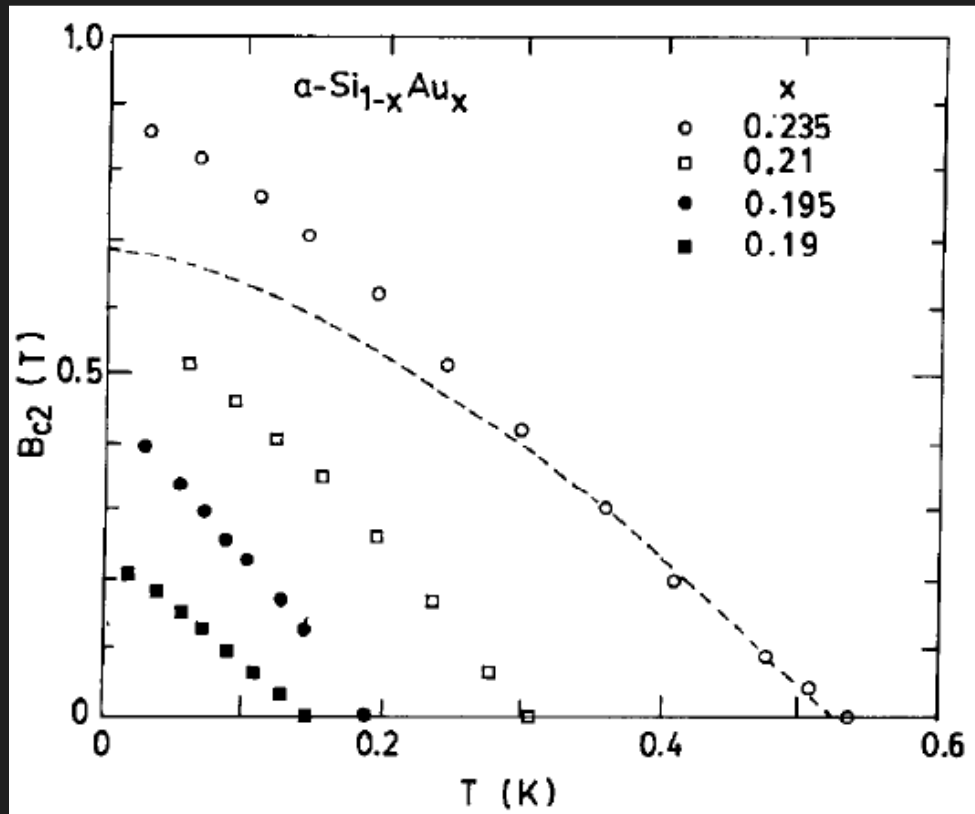


Fisher, Fisher, Huse PRB ('91)
Blatter, et al. RMP ('94)
Kwok, et al. Rep. Prog. Phys. ('16)

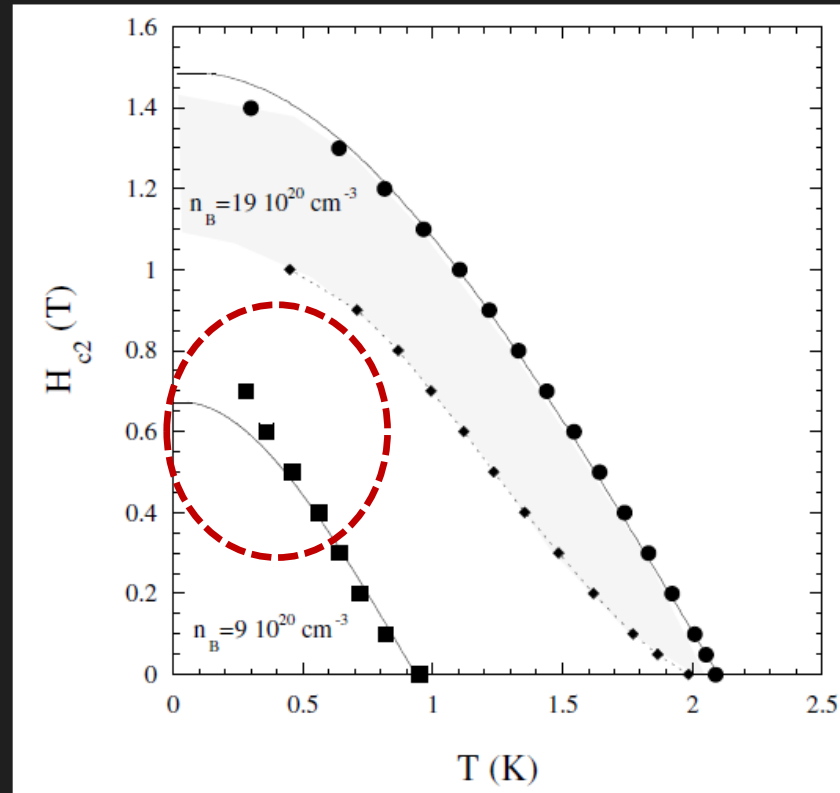
In/InO_x composite films (10 nm thick)



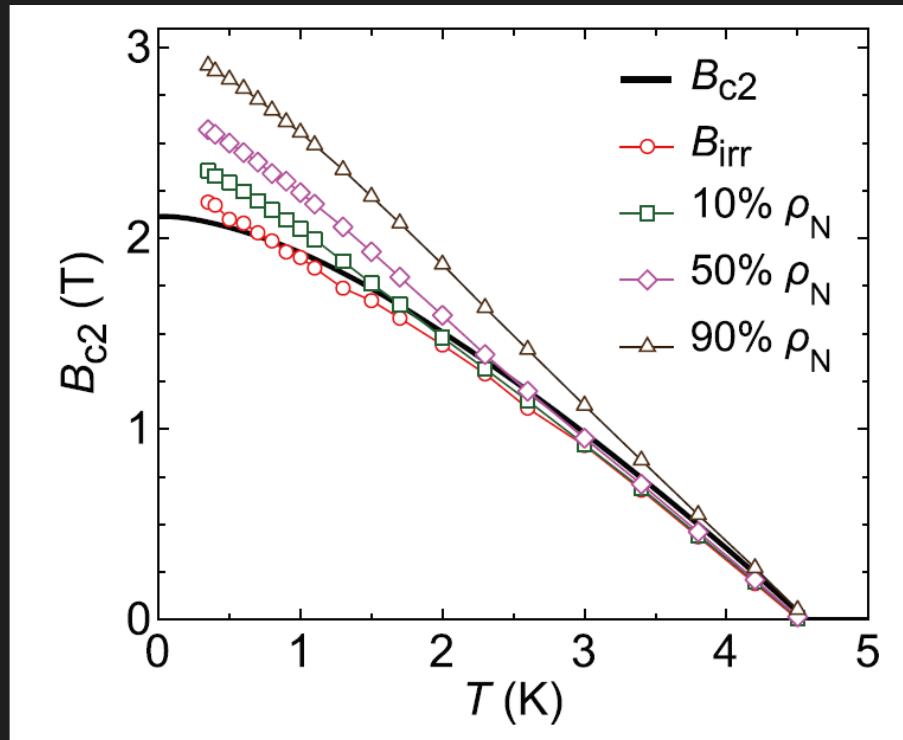
a-Si_{1-x}Au_x films (100-200nm)



B-doped diamond (bulk)



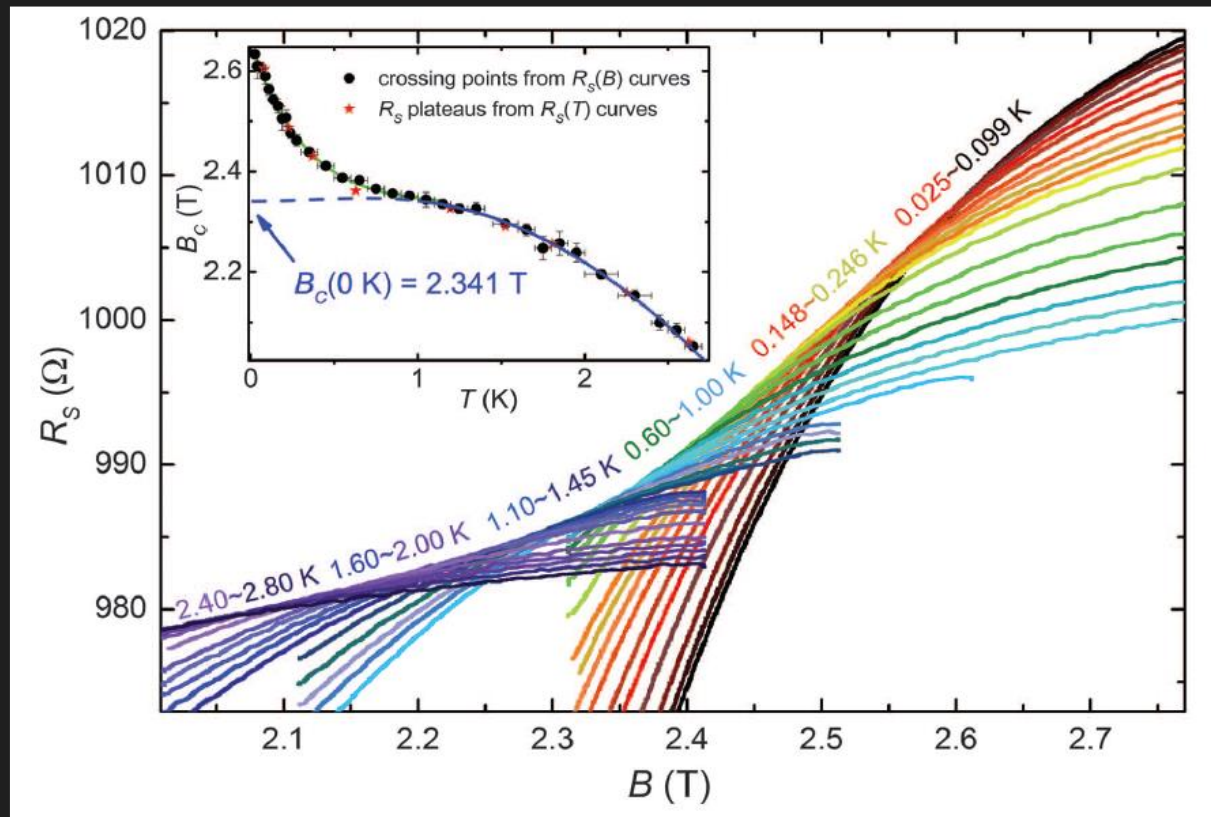
AgSnS₂ (bulk)



Quantum Griffiths singularity of superconductor-metal transition in Ga thin films

Ying Xing,^{1*} Hui-Min Zhang,^{2*} Hai-Long Fu,^{1*} Haiwen Liu,^{1,4*} Yi Sun,¹ Jun-Ping Peng,²
 Fa Wang,^{1,4} Xi Lin,^{1,4,†} Xu-Cun Ma,^{2,3,4,†} Qi-Kun Xue,^{3,4} Jian Wang,^{1,4,†} X. C. Xie^{1,4}

Science 350, 542 (2015)



Low-T anomaly of $B_{c2}(T)$

Tenhover et al. ('81)

Okuma et al. ('83)

Hebard, Paalanen ('84)

Graybeal, Beasley ('84)

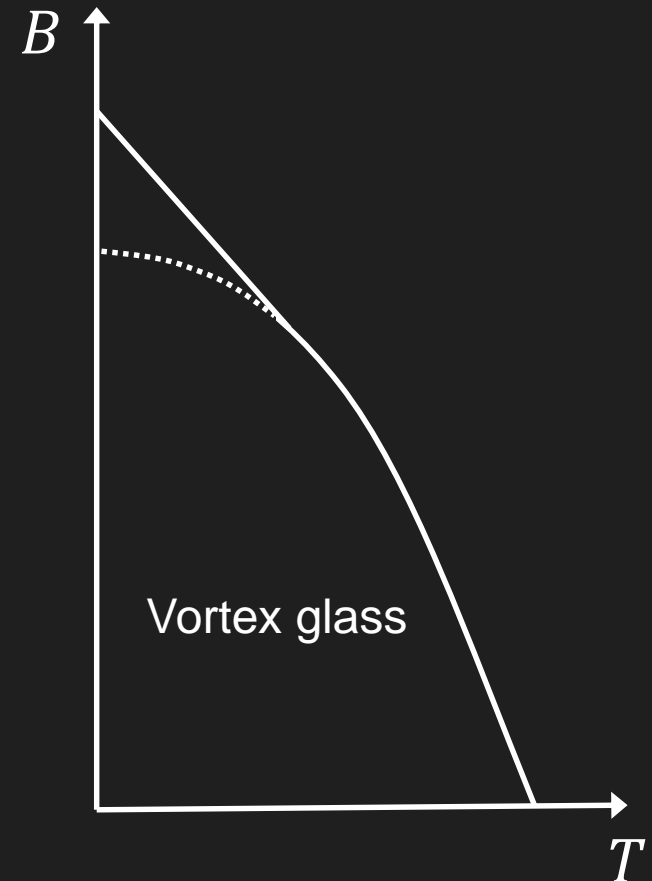
Furubayashi et al. ('85)

Nordström et al. ('93)

Bustarret et al. ('04)

Ren et al. ('13)

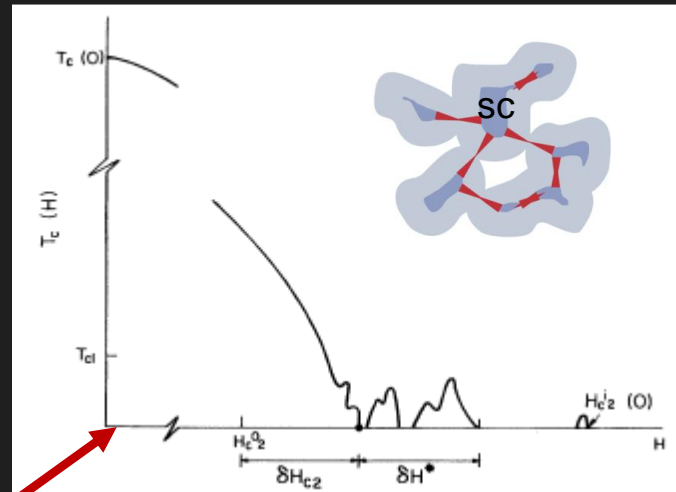
Xing et al. ('15)



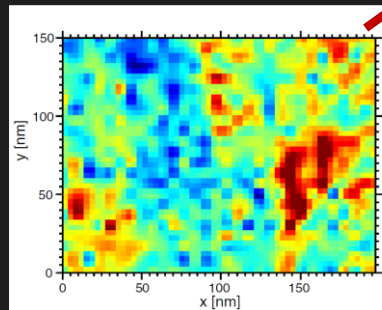
Mesoscopic fluctuations scenario

Spivak & Zhou, PRL ('95)

Mesoscopic sample



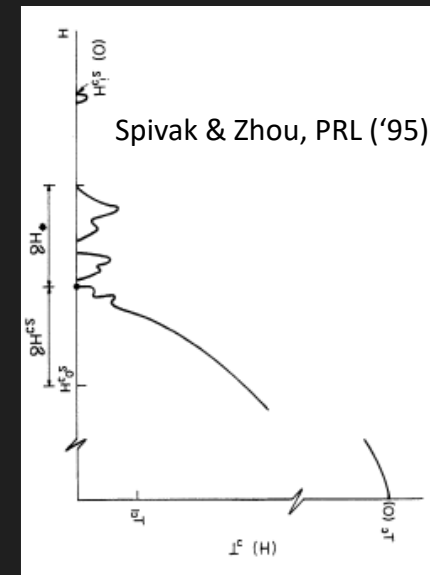
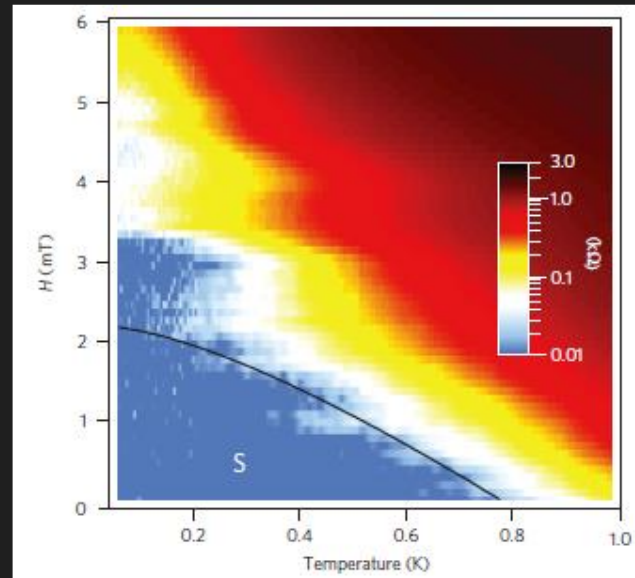
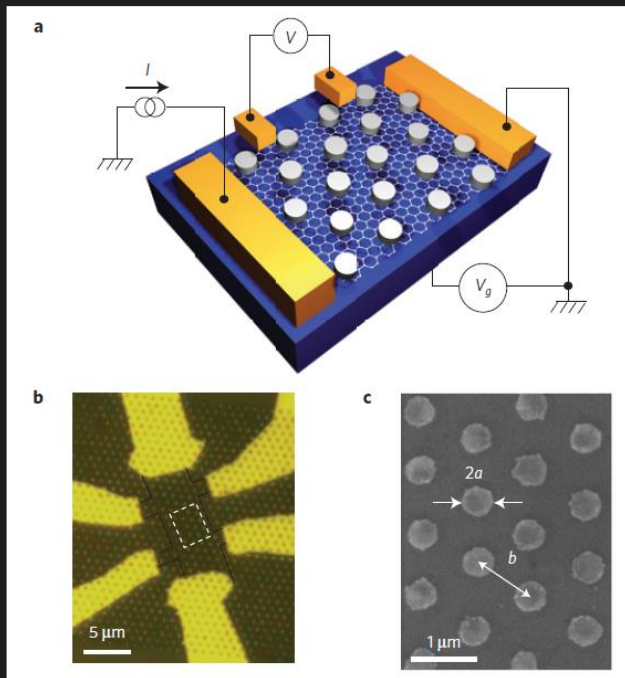
STM map of $\Delta(r)$



B.S. et al, PRL ('08)

Collapse of superconductivity in a hybrid tin-graphene Josephson junction array

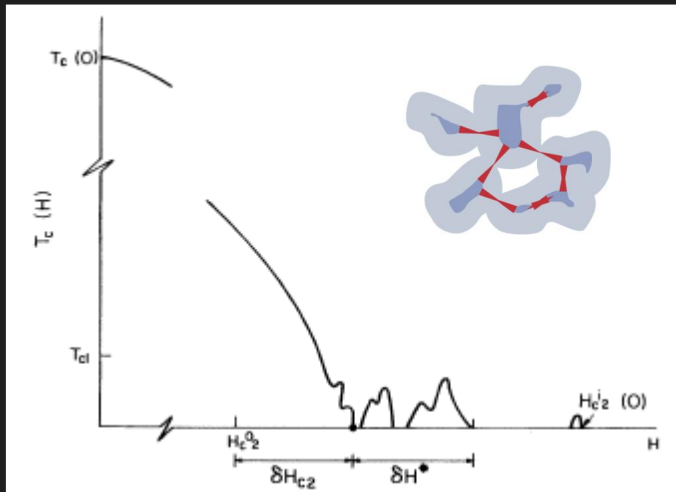
Zheng Han^{1,2}, Adrien Allain^{1,2}, Hadi Arjmandi-Tash^{1,2}, Konstantin Tikhonov^{3,4}, Mikhail Feigel'man^{3,5}, Benjamin Sacépé^{1,2} and Vincent Bouchiat^{1,2*}



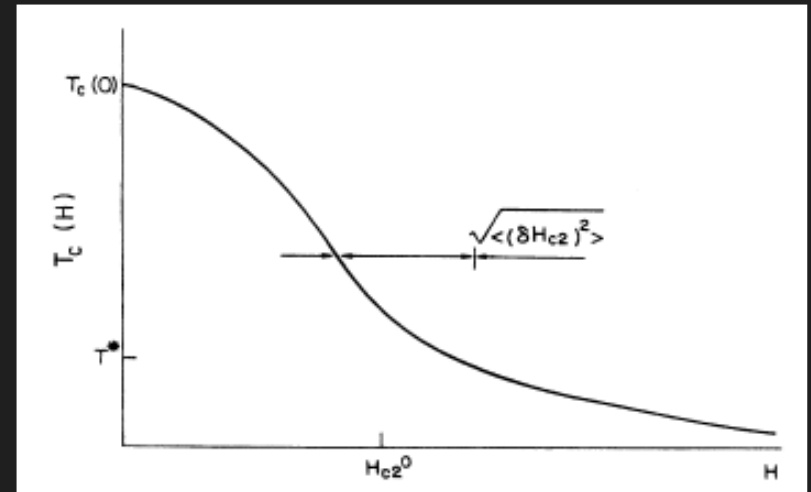
Mesoscopic fluctuations scenario

Spivak & Zhou, PRL ('95)

Mesoscopic sample



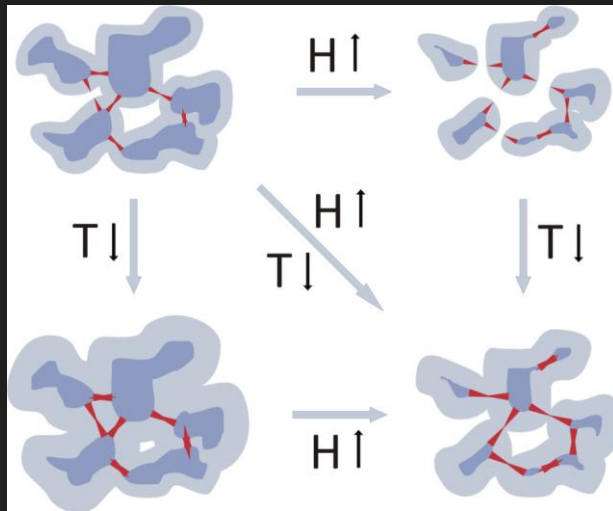
Bulk sample



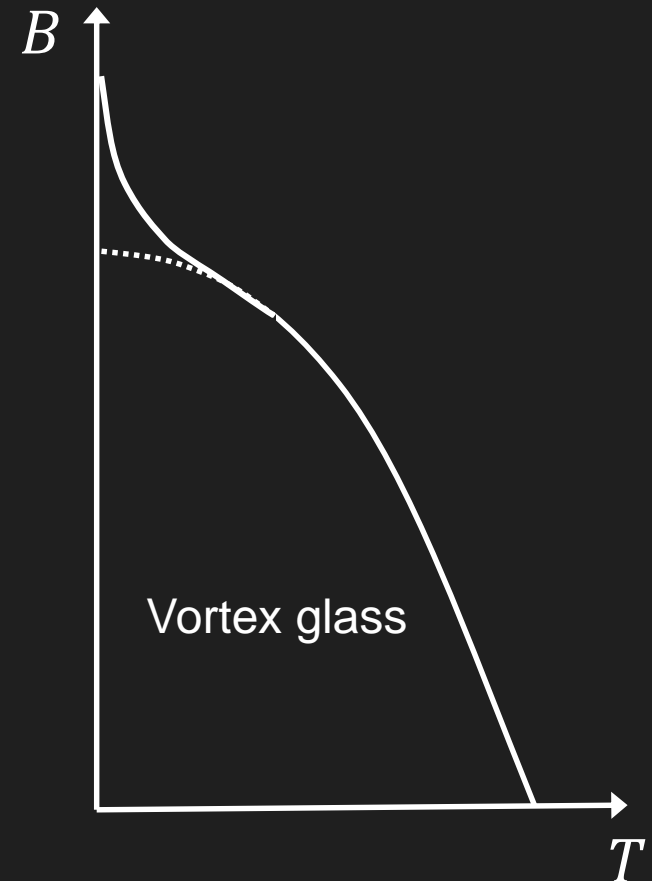
Predict exponential decrease of $\langle j_c \rangle \sim \exp\left(-\frac{R_0}{L_H} - \frac{R_0}{L_T}\right)$

Mesoscopic fluctuations scenario

Galitski & Larkin, PRL ('01)



- $B \nearrow$: decrease of SC island size
- $T \searrow$: increase of SC proximity effect

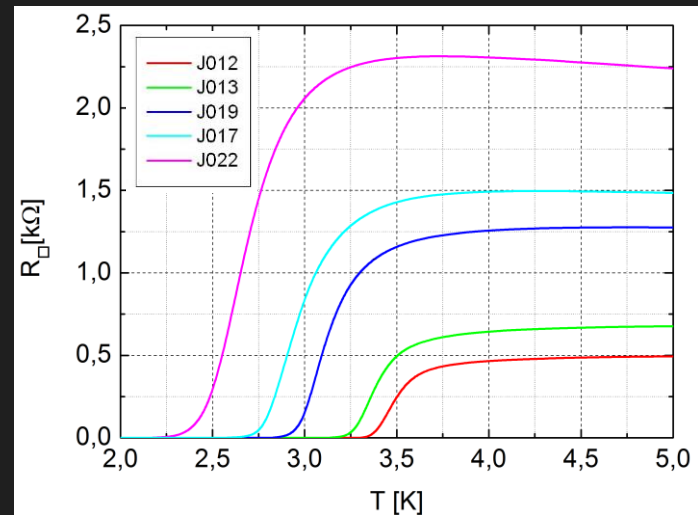
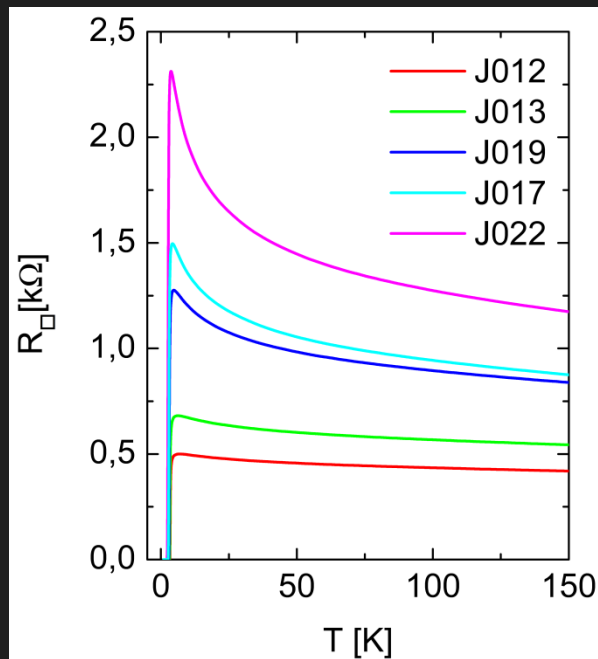


Predict exponential increase of $B_{c2}(T)$ and exp. suppression of $j_c(B)$!

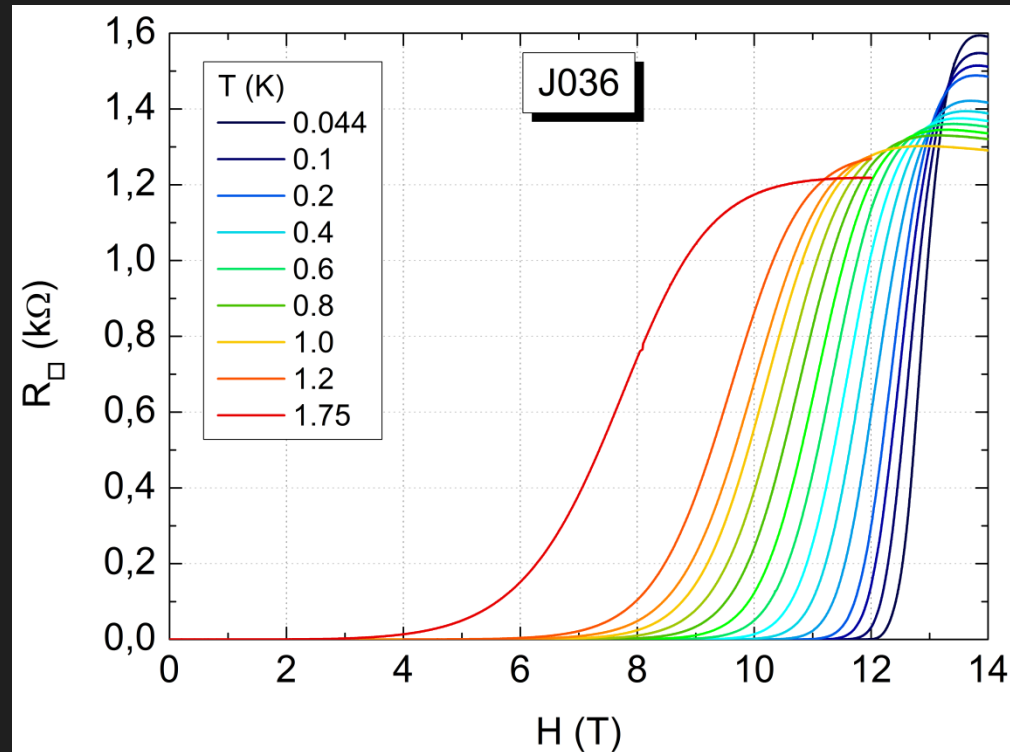
Critical current

Moderately disordered amorphous indium oxide (InO)

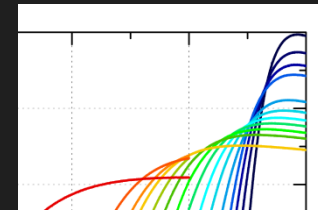
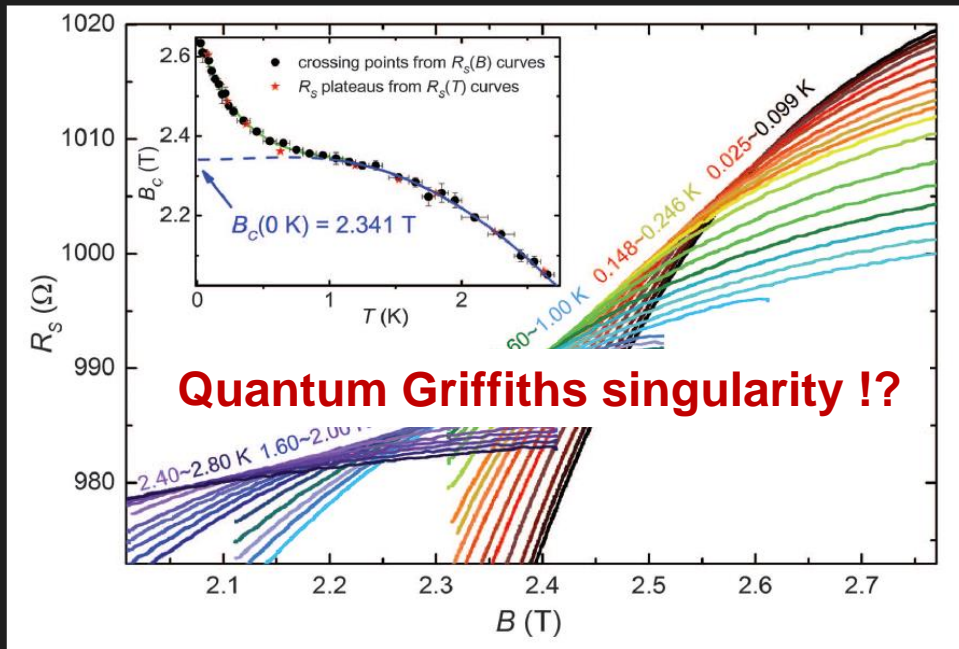
- 30-60 nm thick
- e-density : $n \sim 10^{21} \text{ cm}^{-3}$
- Disorder : $k_F l_e \sim 0.3 - 0.4$



Moderately disordered amorphous indium oxide (InO)

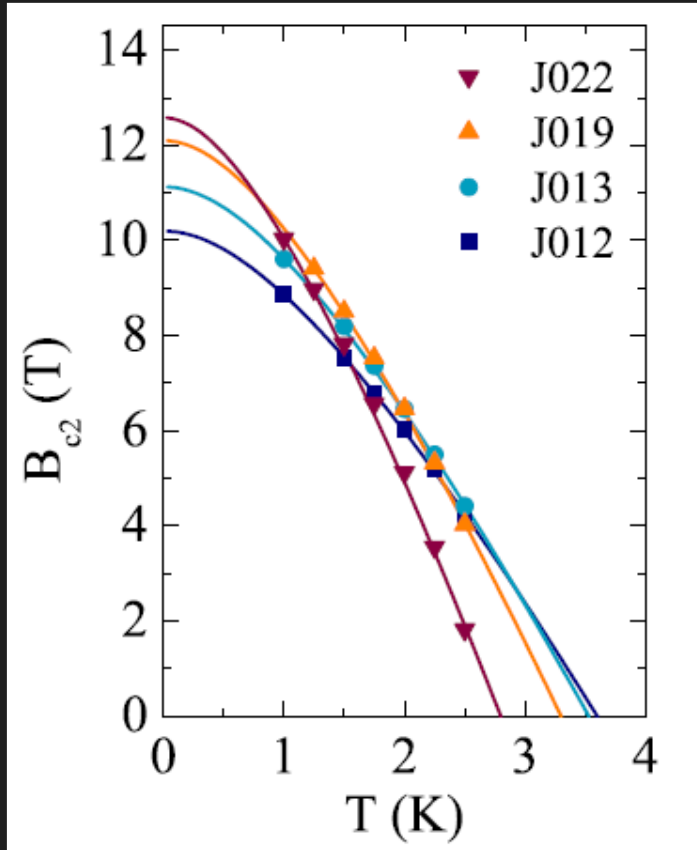


Moderately disordered amorphous indium oxide (InO)



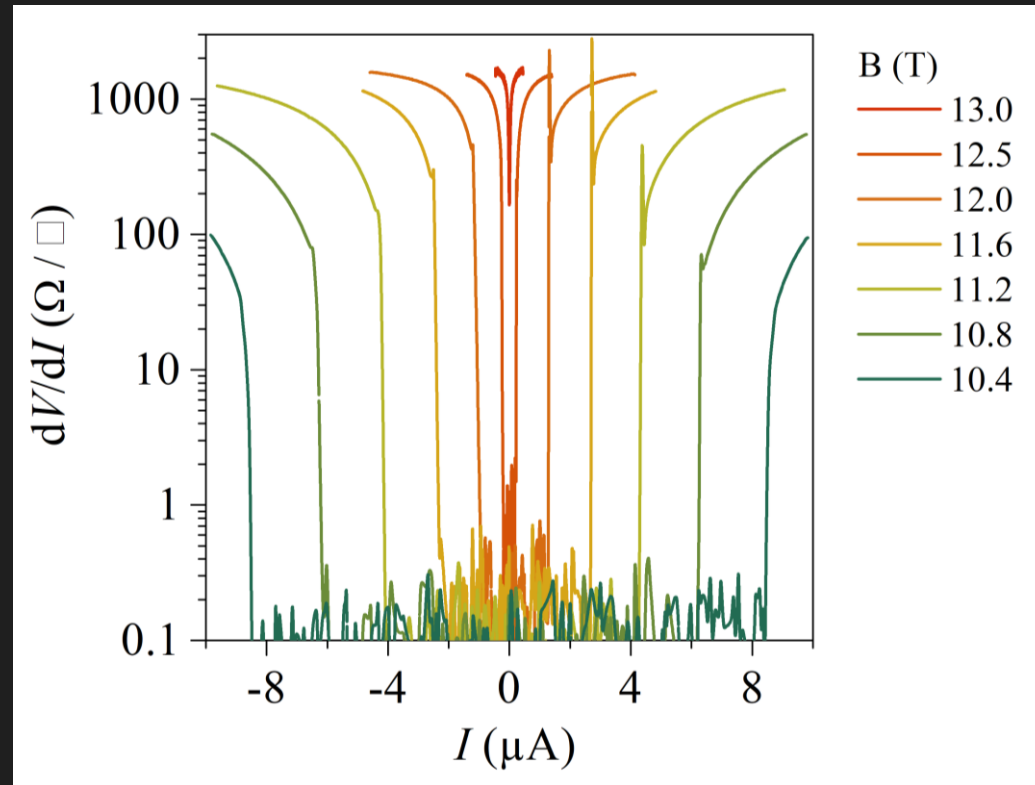
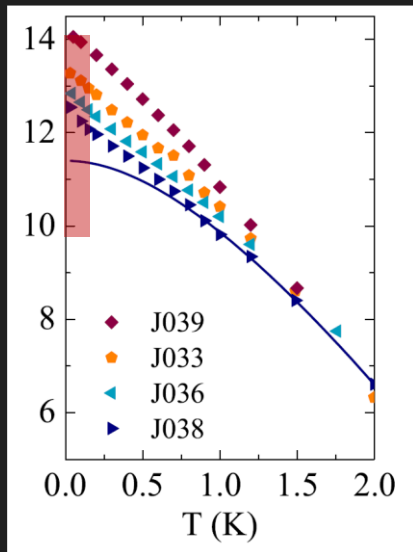
Science 350, 542 (2015)

Linear T -dependence of $B_{c2}(T)$

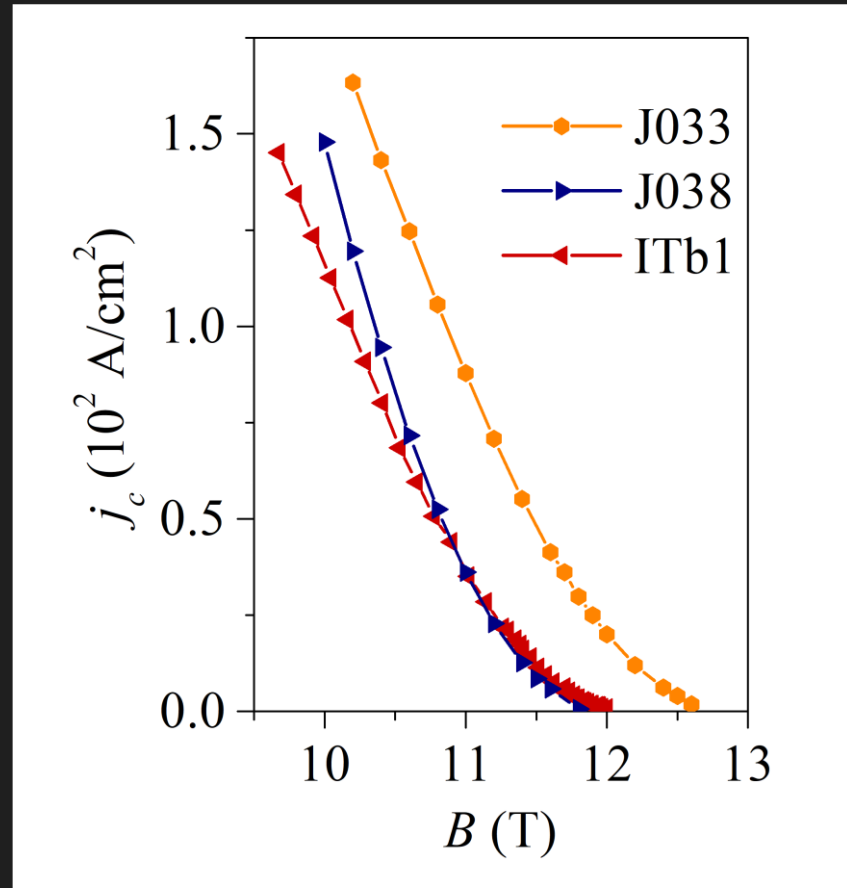
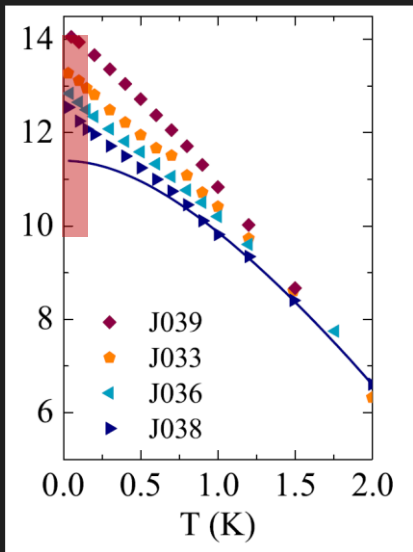


Critical current measurements

$T \simeq 0.03 \text{ K}$

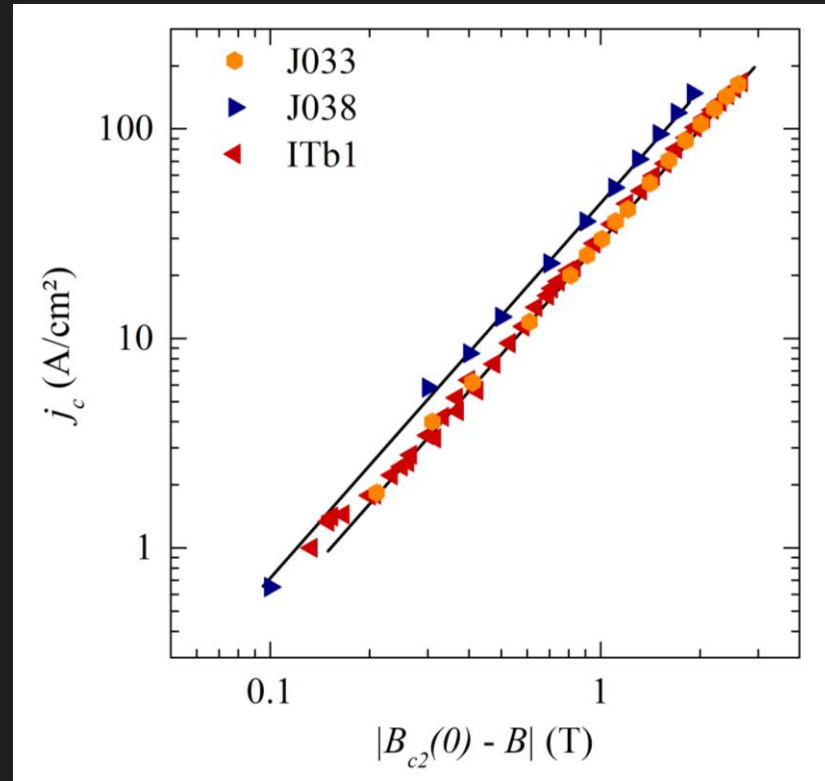


Critical current measurements



Critical current measurements

$T \simeq 0.03 \text{ K}$



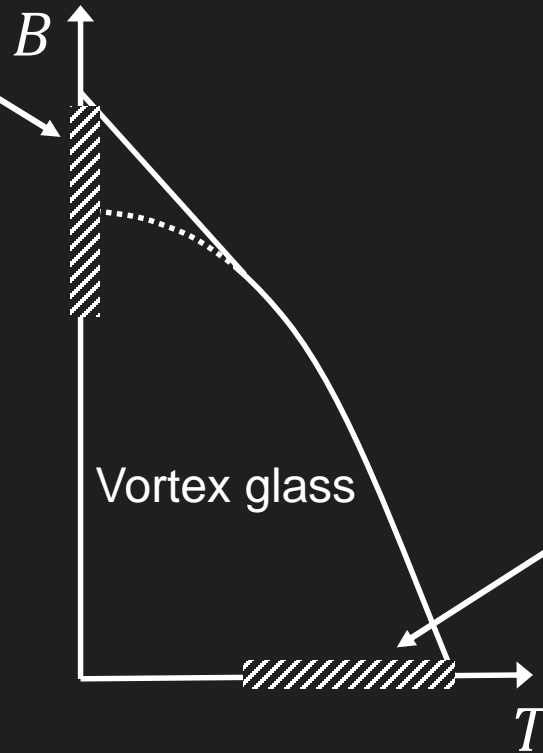
$$J_c(T \sim 0) \propto (B_{c2}(0) - B)^\alpha \quad \alpha \simeq 1.5 - 1.6$$

Mean-field depairing current ?

Near quantum transition:

$$J_c \propto |B - B_{c2}|^{3/2} \propto \frac{\rho_s}{\xi_{GL}}$$

$$\begin{cases} \rho_s \propto |B - B_{c2}| \\ \xi_{GL} \propto |B - B_{c2}|^{-1/2} \end{cases} \quad ?$$



Near thermal transition:

$$\begin{cases} \rho_s \propto |T - T_c| \\ \xi_{GL} \propto |T - T_c|^{-1/2} \end{cases}$$

$$J_c \propto \frac{\rho_s}{\xi_{GL}} \propto |T - T_c|^{3/2}$$

Ginzburg-Landau

$$F = \alpha |\Delta(\mathbf{r})|^2 + \beta |\Delta(\mathbf{r})|^4 + \gamma \left| \left(-i\nabla - \frac{2e}{\hbar c} \mathbf{A}(\mathbf{r}) \right) \Delta(\mathbf{r}) \right|^2$$

$$j_c \propto \rho_s / \xi_{GL}$$

From the free energy

$$\mathbf{j} = -c \frac{\partial F}{\partial \mathbf{A}}$$

$$\mathbf{j} = \gamma \frac{2e}{\hbar} |\Delta(\mathbf{r})|^2 \mathbf{A}$$

$$\rho_s = \frac{\hbar c}{2e} \gamma |\Delta(\mathbf{r})|^2$$

From London equation

$$\mathbf{j} = -4\rho_s \frac{e^2 \mathbf{A}}{\hbar^2 c}$$

Ginzburg-Landau

$$F = \alpha |\Delta(\mathbf{r})|^2 + \beta |\Delta(\mathbf{r})|^4 + \gamma \left| \left(-i\nabla - \frac{2e}{\hbar c} \mathbf{A}(\mathbf{r}) \right) \Delta(\mathbf{r}) \right|^2$$

$$|\Delta|^2 = \frac{\alpha}{2\beta}$$

$$\alpha = v \left[\ln \frac{T}{T_{c0}} + \psi \left(\frac{1}{2} + \frac{eDB}{2\pi cT} \right) - \psi \left(\frac{1}{2} \right) \right]$$

$$\alpha = \xrightarrow{T \rightarrow 0} v \left(1 - \frac{B}{B_c} \right)$$

$$\rho_s = \frac{\hbar c}{2e} \gamma |\Delta(\mathbf{r})|^2 = \frac{12}{\pi} \rho_{s0} \left(1 - \frac{B}{B_{c2}(0)} \right)$$

Critical current

$$j_c \propto \rho_s / \xi_{GL}$$

Superfluid stiffness

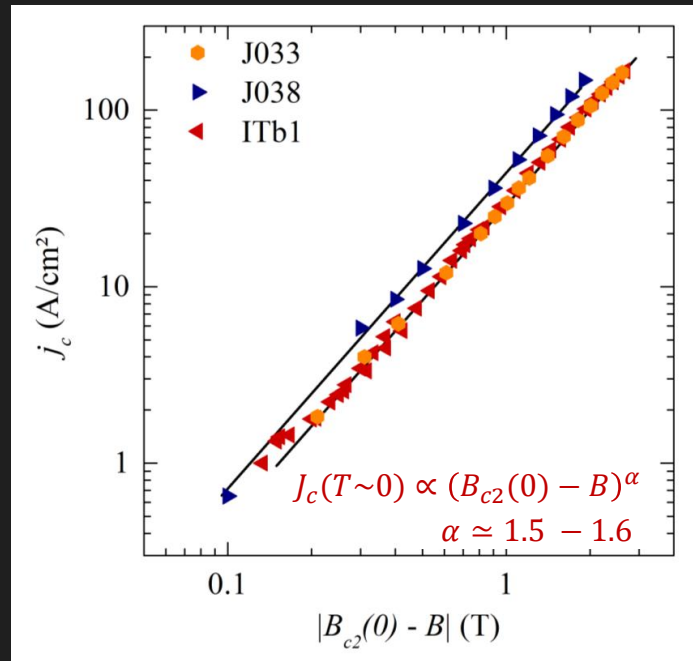
$$\rho_s \propto |\Delta|^2 \sim \left(1 - \frac{B}{B_{c2}(0)}\right)$$

Coherence length

$$\xi_{GL} \sim \frac{v_F}{\Delta}$$

$$J_c \propto \frac{\rho_s}{\xi_{GL}} \sim \left(1 - \frac{B}{B_{c2}(0)}\right)^{3/2}$$

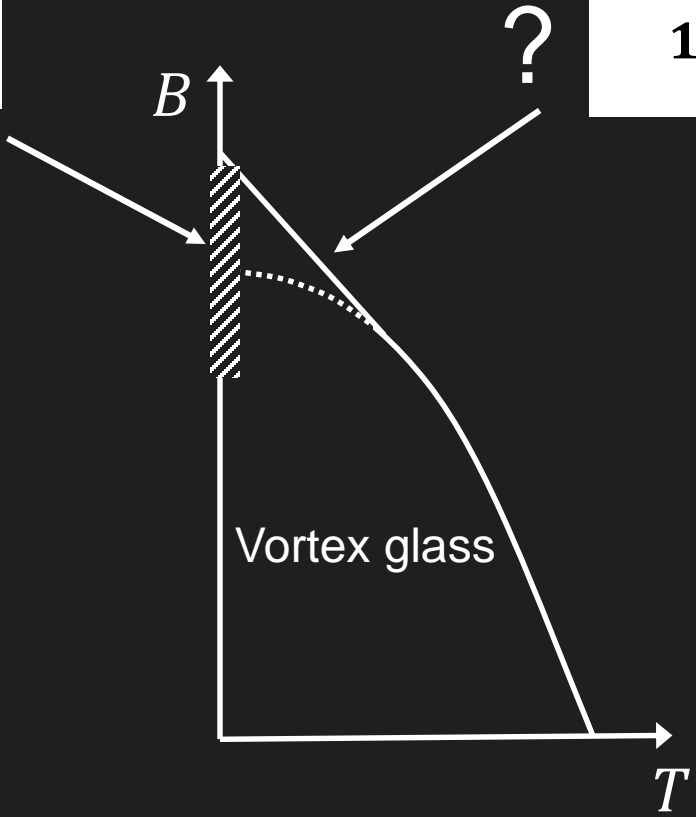
Mean-field scaling of the critical current



$$J_c \propto \frac{\rho_s}{\xi_{GL}} \sim \left(1 - \frac{B}{B_{c2}(0)}\right)^{3/2}$$

$$\rho_s \sim \left(1 - \frac{B}{B_{c2}(0)}\right)$$

$$1 - \frac{B_{c2}(T)}{B_{c2}(0)} \sim T$$



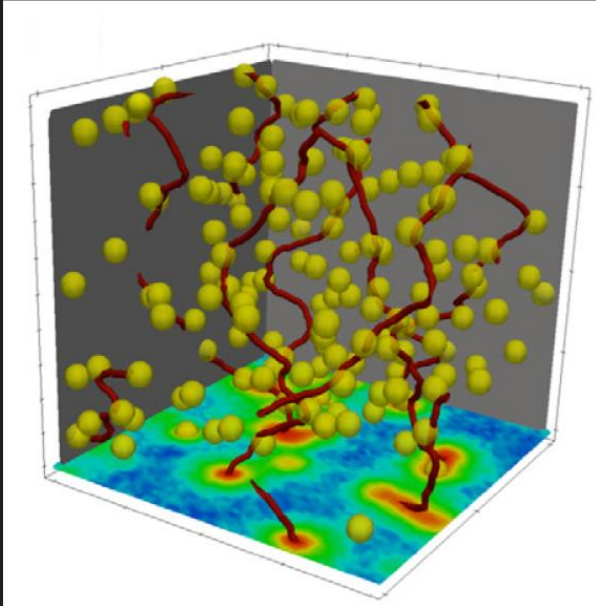
Low- T anomaly of B_{c2}

For bulk crystal $T_c(B)$ is given by :

$$\delta\rho_s(B, T_c) = \epsilon\rho_s(B, 0)$$

Similar to the Lindemann criterion for the melting of bulk crystal

Low- T thermal fluctuations of the vortex glass



Kwok et al. Rep. Prog. Phys. ('16)

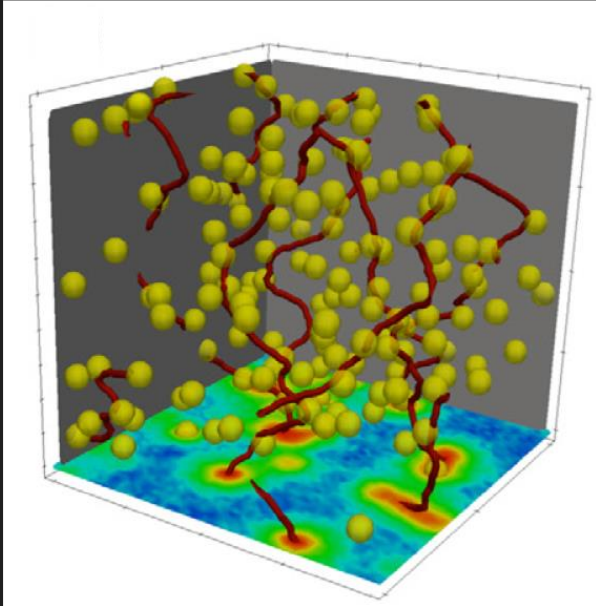
Correction to the superfluid density :

$$\delta\rho_s(T, B) = -C \frac{\hbar\sigma_n}{e^2} \frac{T^2}{3\pi\rho_s(B)a_0}$$

Valid for $T \ll T_c$ and $\delta\rho_s(T, B) \ll \rho_s(0, B)$

For details see Feigel'man talk

Low- T thermal fluctuations of the vortex glass



Kwok et al. Rep. Prog. Phys. ('16)

Correction to the superfluid density :

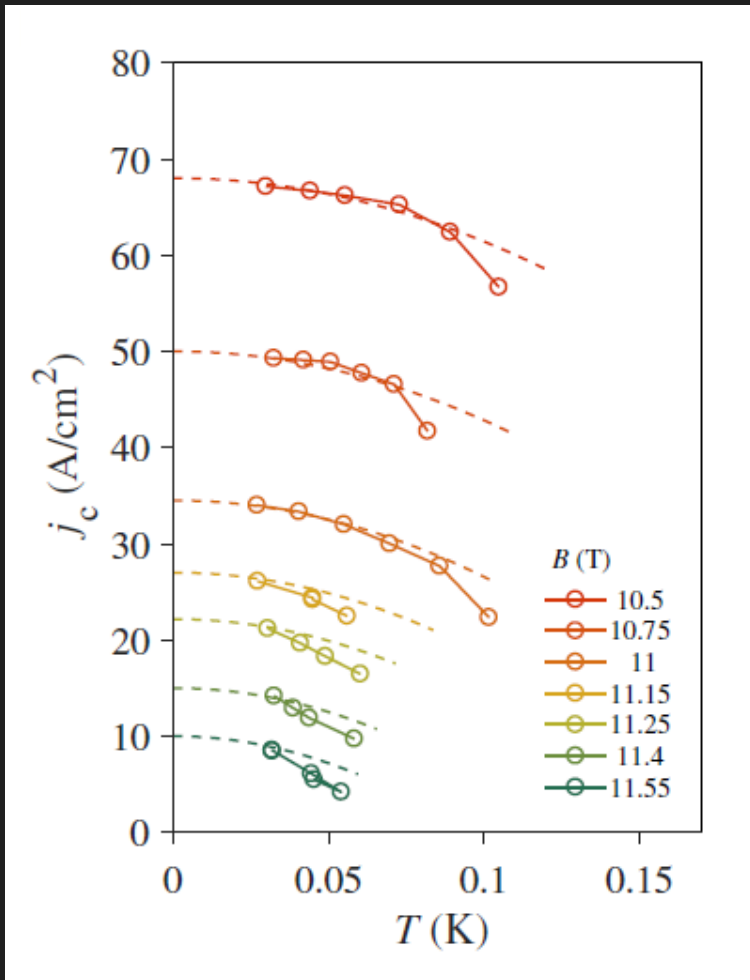
$$\delta\rho_s(T, B) = -C \frac{\hbar\sigma_n}{e^2} \frac{T^2}{3\pi\rho_s(B)a_0}$$

Valid for $T \ll T_c$ and $\delta\rho_s(T, B) \ll \rho_s(0, B)$

→ correction to critical current :

$$\delta j_c^{GL}(T, B) \propto \frac{\delta\rho_s(T, B)}{\xi_{GL}} \propto \frac{T^2}{\sqrt{B_{c2}(0) - B}}$$

Low- T thermal fluctuations of the vortex glass



Correction to the superfluid density :

$$\delta\rho_s(T, B) = -C \frac{\hbar\sigma_n}{e^2} \frac{T^2}{3\pi\rho_s(B)a_0}$$

Valid for $T \ll T_c$ and $\delta\rho_s(T, B) \ll \rho_s(0, B)$

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Low- T anomaly of B_{c2}

☑ **Bulk crystal** : $T_c(B)$ is given by

$$\delta\rho_s(B, T_c) = \epsilon\rho_s(B, 0)$$

Similar to the Lindemann criterion for the melting of bulk crystal

$$\delta\rho_s(T, B) = -C \frac{\hbar\sigma_n}{e^2} \frac{T^2}{3\pi\rho_s(B)a_0} = \epsilon\rho_s(B)$$

$$\rho_s(B) \propto \left(1 - \frac{B_{c2}(T)}{B_{c2}(0)}\right) \propto T$$

☑ **Thin films** : Generalized BKT transition

$$\rho_s(B, T_{BKT}) = \frac{\chi}{d} T_{BKT}$$

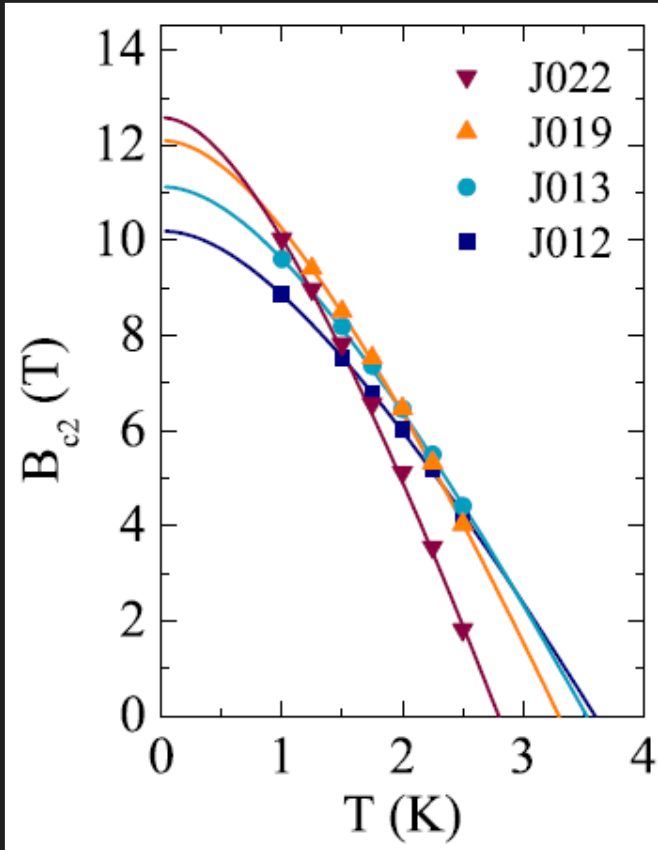
$$\chi^{-1} \sim 1.5 - 2$$

T. Schneider, and A. Schmidt ('92)

$$1 - \frac{B_{c2}(T)}{B_{c2}(0)} \propto \frac{T}{d}$$

Thickness

Linear T -dependence of $B_{c2}(T)$

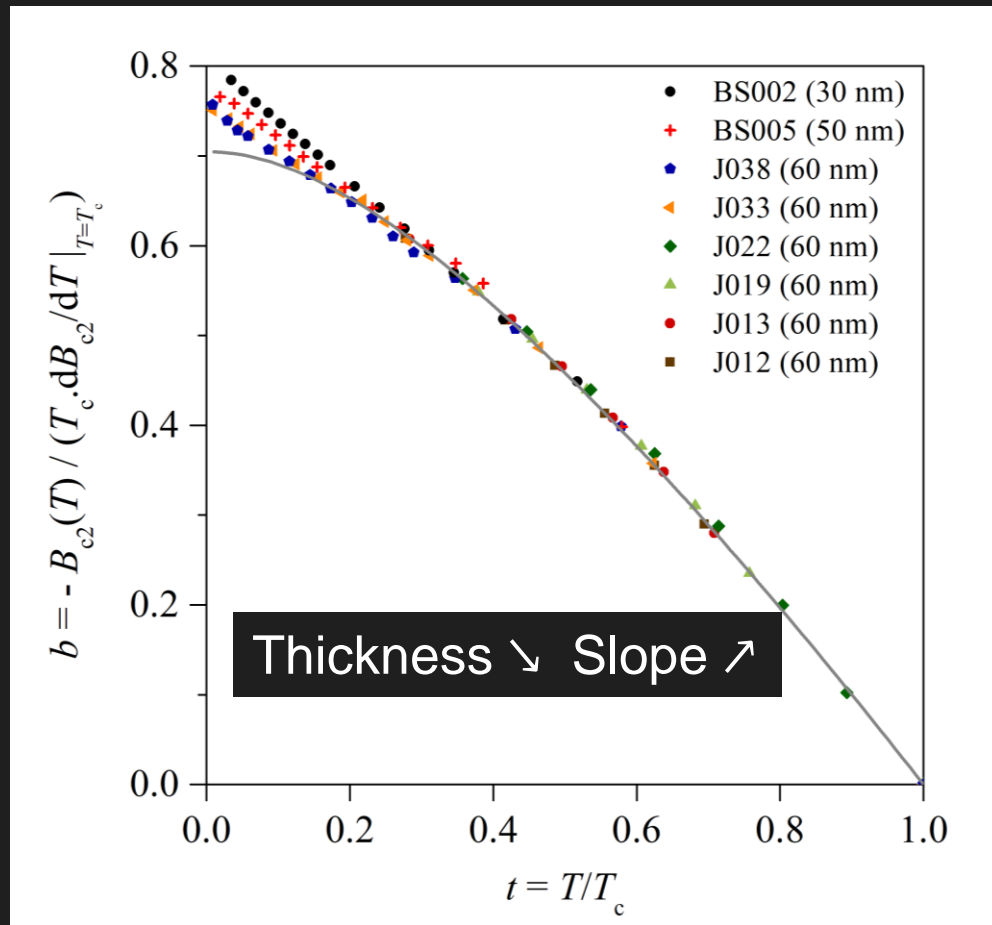


$$\left. \frac{dB_{c2}}{dT} \right|_{T \rightarrow T_c} \propto -\frac{1}{D}$$

$$B_{c2}(0) = \frac{\phi_0}{2\pi\xi^2} \propto \frac{1}{D}$$

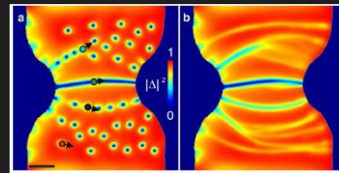
$$b = - \frac{B_{c2}(T)}{\left(T_c \frac{dB_{c2}}{dT} \Big|_{T \rightarrow T_c} \right)} \quad \text{versus} \quad T/T_c$$

WHH ('66)



De-pairing vs. de-pinning

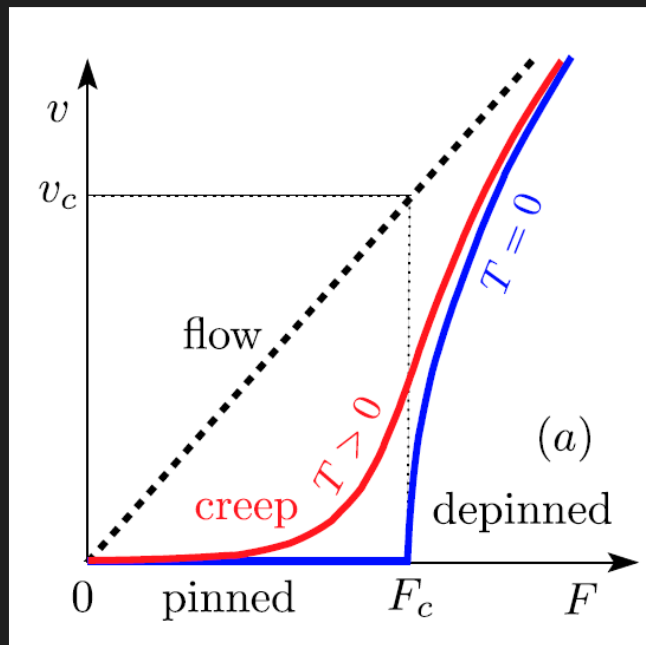
$$J_c \propto \frac{\rho_s}{\xi_{GL}}$$



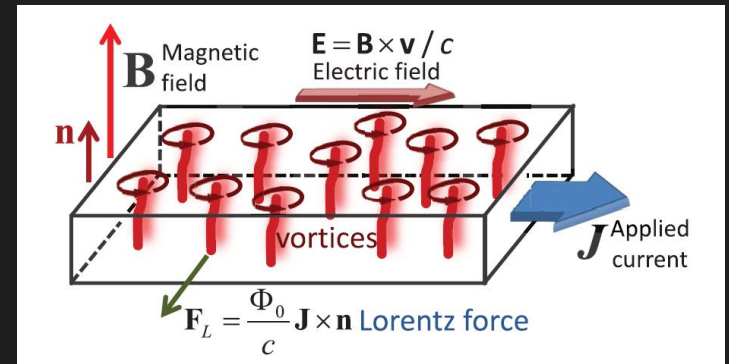
E. Zeldov ('17)

Weak vs strong pinning

WEAK pinning



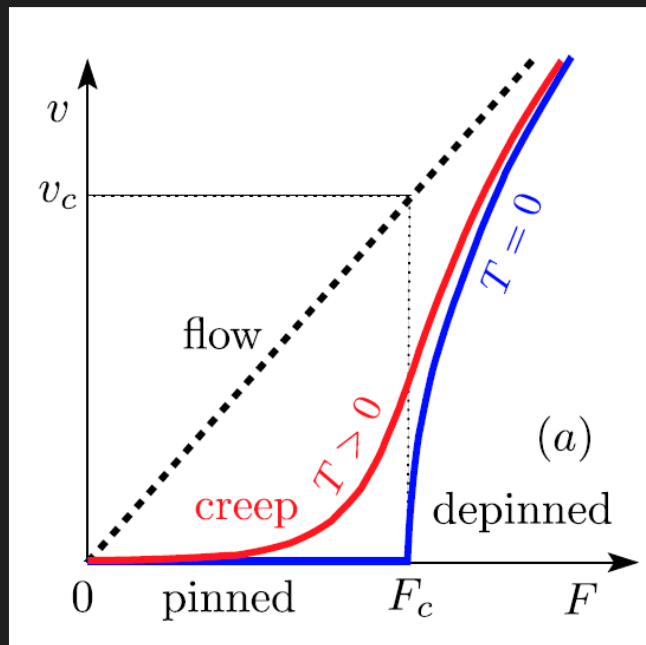
Pinning collapses beyond F_c



Kwok et al. Rep. Prog. Phys. ('16)

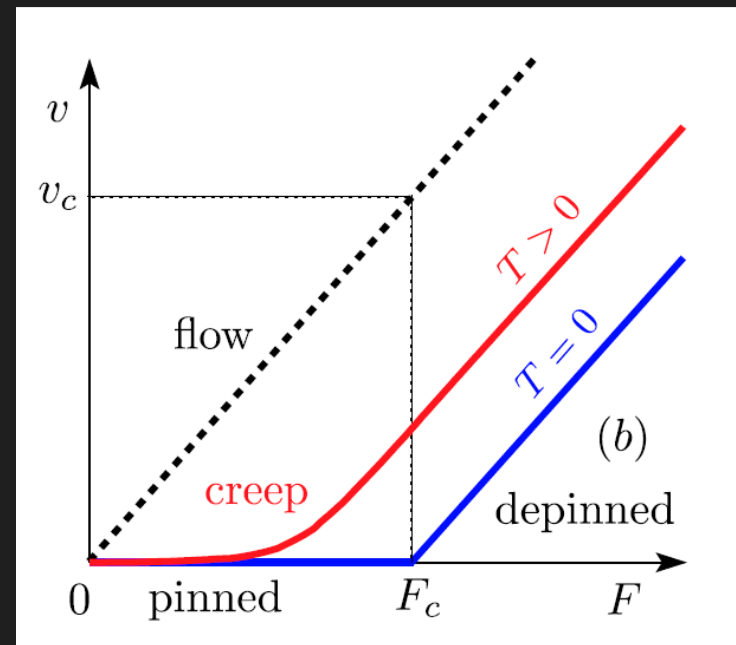
Weak vs strong pinning

WEAK pinning



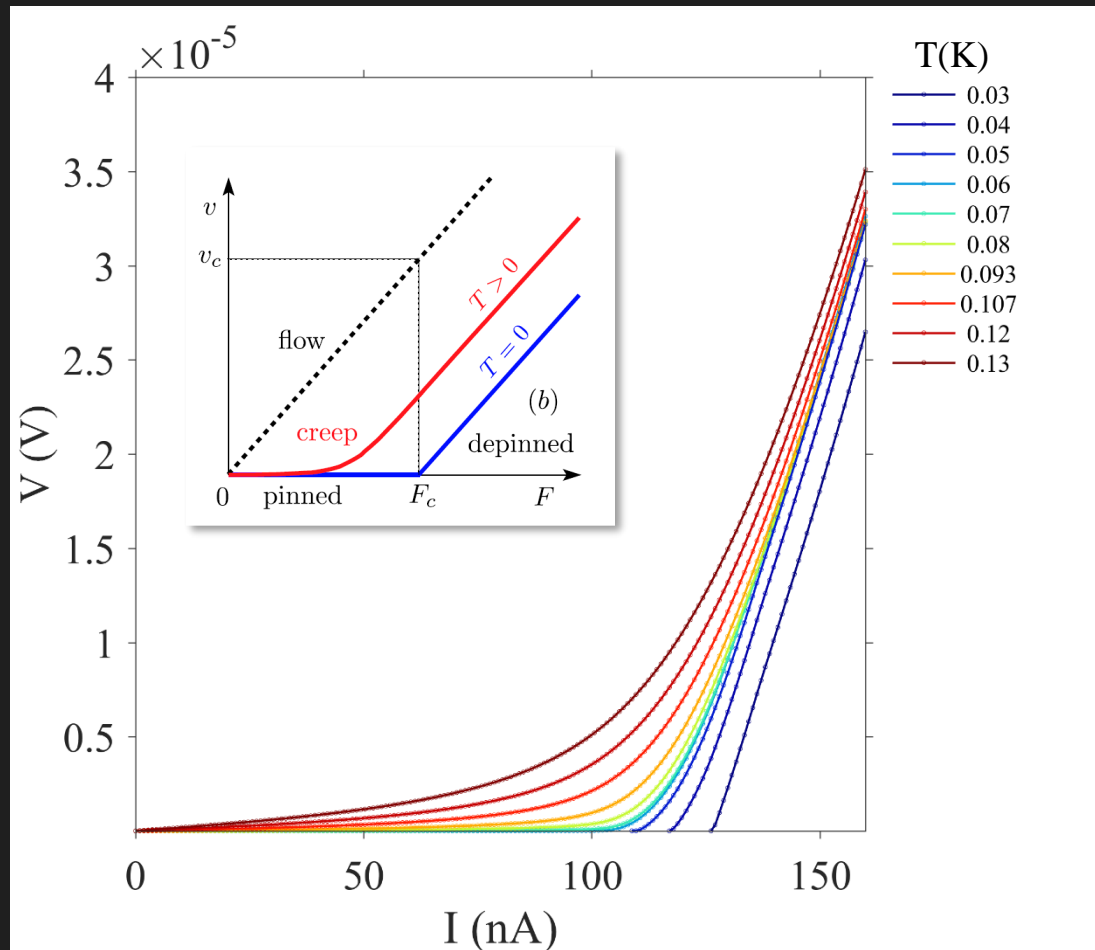
Pinning collapses beyond F_c

STRONG pinning



Vortex creep above F_c

Strong pinning !

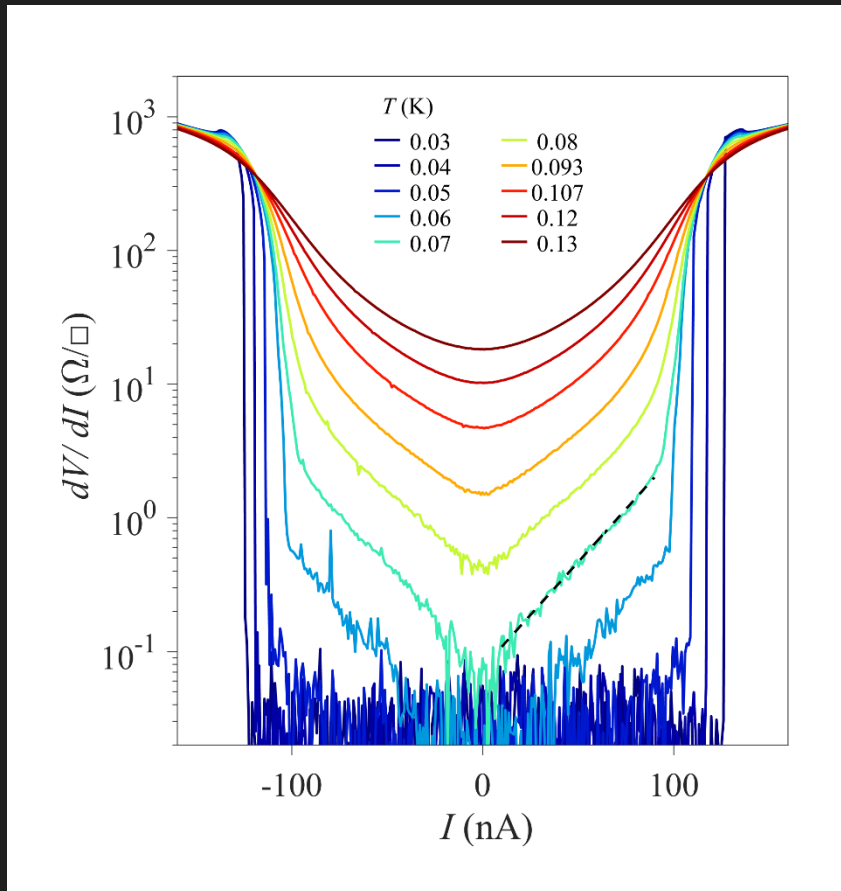


Strnad, Hempstead, Kim ('64-'65)

Xiao, Andrei, Paltiel, Zeldov, Shuk, Greenblatt ('02)

De-pinning transition

$B = 11.25 \text{ T}$

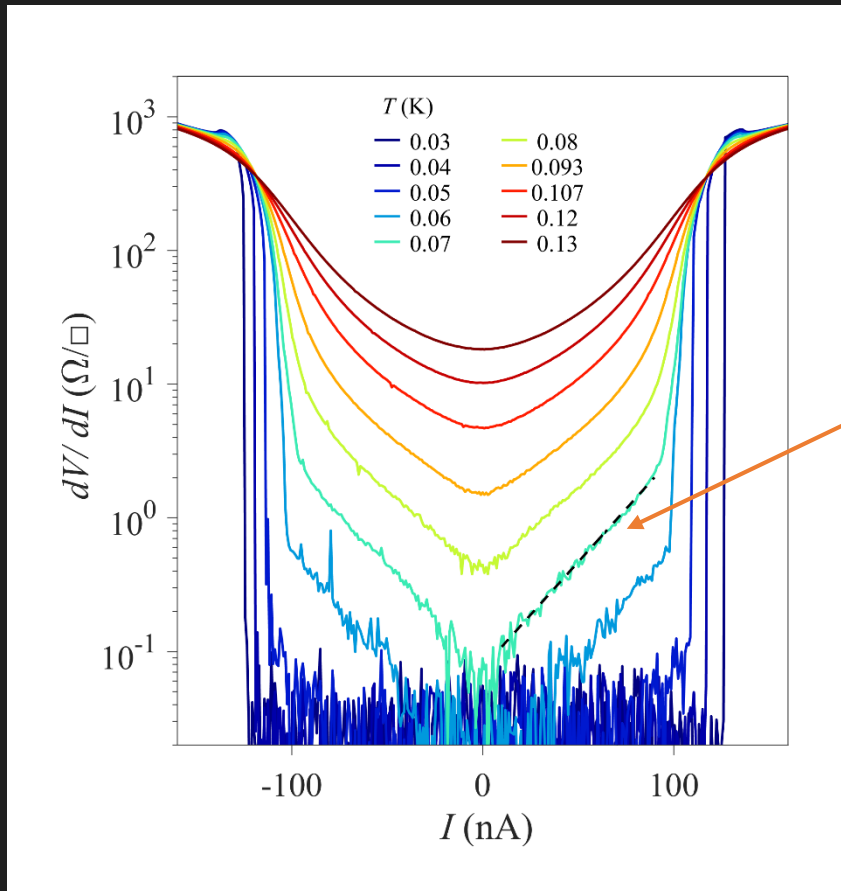


☑ Jump of several orders of dV/dI :

➔ **collective de-pinning !?**

De-pinning transition

B = 11.25 T



☑ Exponential increase of dV/dI

$$R(T, B, j) = R_0 e^{-U(B, j)/T}$$

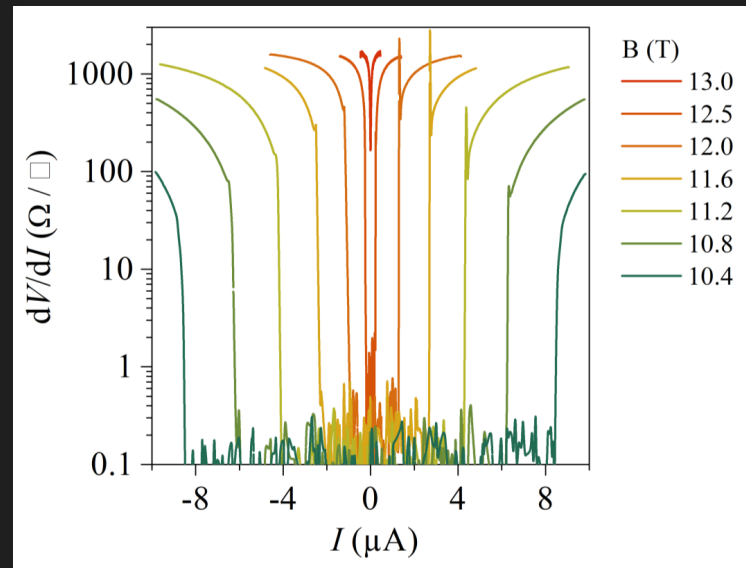
$$U(B, j) = U(B) \left(1 - \frac{j}{j_1} \right)$$

De-pairing vs. de-pinning

- ✓ Large resistance (~ kOhms / \square)
- ✓ Signatures of strong pinning in IV's
- ✓ Collective depinning



$$j_c^{de-pinning}$$



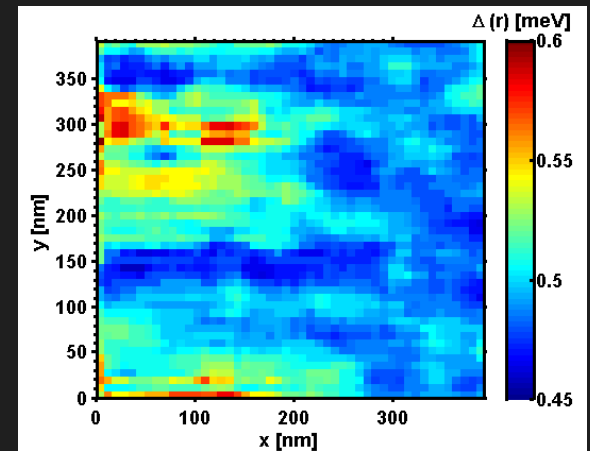
De-pairing vs. de-pinning

☑ **Mean-field scaling** $j_c \sim |B - B_{c2}|^{3/2}$

De-pairing vs. de-pinning

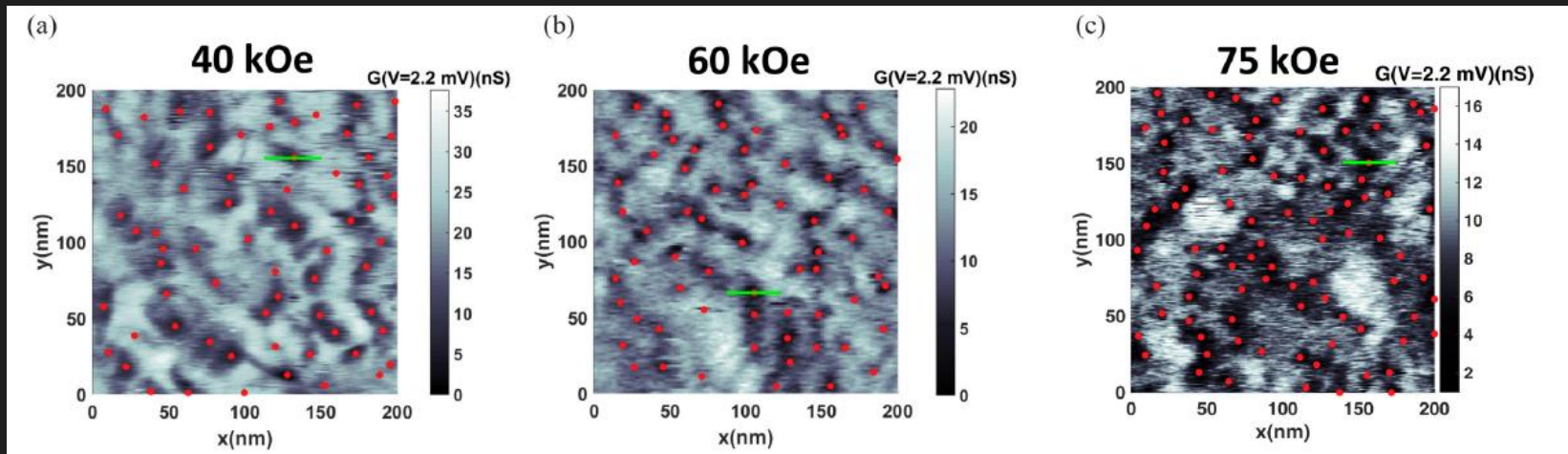
- ✓ Mean-field scaling $j_c \sim |B - B_{c2}|^{3/2}$
- ✓ Strong spatial fluctuations of $\Delta(r)$

InOx films ($B=0$)



B. Sacépé, et al., *Nat. Phys.* ('11)

NbN films



P. Raychaudhuri *PRB* ('17)

De-pairing vs. de-pinning

- ✓ **Mean-field scaling** $j_c \sim |B - B_{c2}|^{3/2}$
- ✓ **Strong spatial fluctuations of $\Delta(r)$**
- ✓ **Similar to columnar defects**

Mkrtchyan & Schmidt JETP ('72)



$$j_c^{de-pinning} = \Gamma j_c^{GL}$$

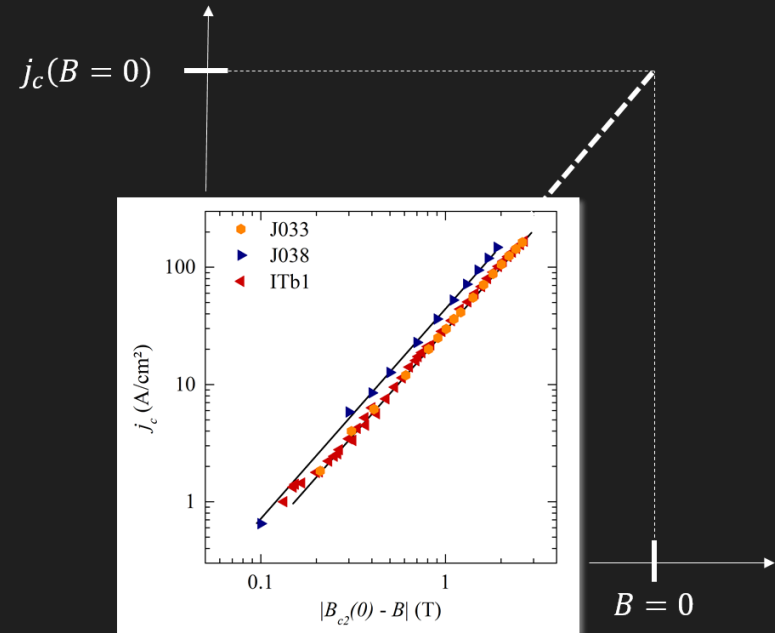
$$(\Gamma < 1)$$

$$J_c^{GL}(B) = J_c^{GL}(0) \left(1 - \frac{B}{B_{c2}(0)} \right)^{3/2}$$

$$J_c^{GL}(0) = \frac{4e\rho_{s0}}{3\sqrt{3}\pi\hbar\xi_{GL}}$$

ρ_{s0} from Yazdani PRL ('13)

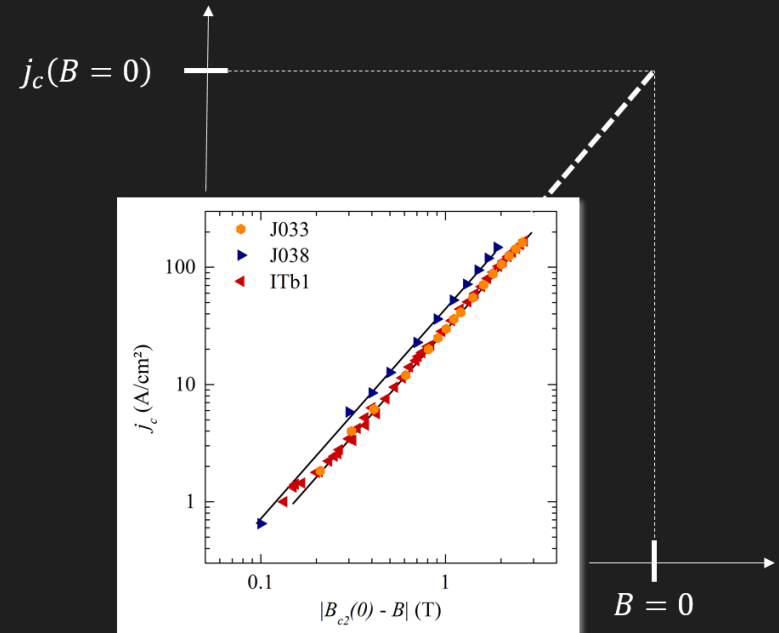
$\xi_{GL} \sim 5$ nm from B.S. PRB ('15)



$$J_c^{GL}(B) = J_c^{GL}(0) \left(1 - \frac{B}{B_{c2}(0)} \right)^{3/2}$$

$$J_c^{GL}(0) \sim 10^4 \text{ A/cm}^2$$

$$J_c(0) \approx (2.5 - 4) \cdot 10^3 \text{ A/cm}^{-2}$$



$$J_c^{de-pinning} = \Gamma J_c^{GL}$$

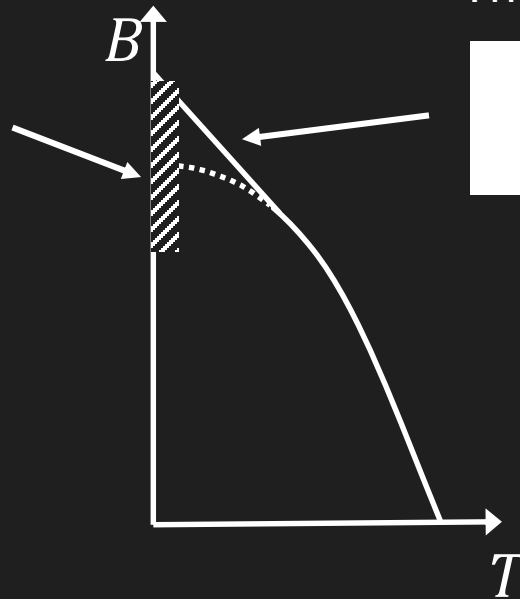
$$\Gamma^{-1} \sim 2 - 4$$

Conclusion

➤ Mean-field scaling $J_c \sim |B - B_{c2}|^{3/2}$

➤ $J_c^{de-pinning} = \Gamma J_c^{GL}$

$$\rho_s \sim \left(1 - \frac{B}{B_{c2}(0)} \right)$$

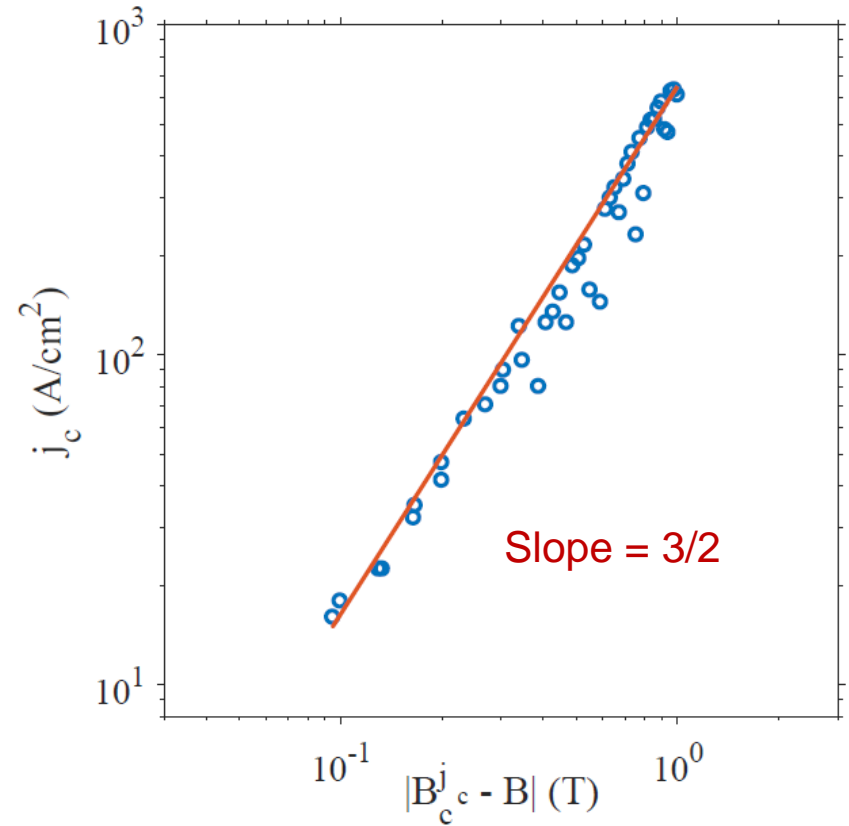
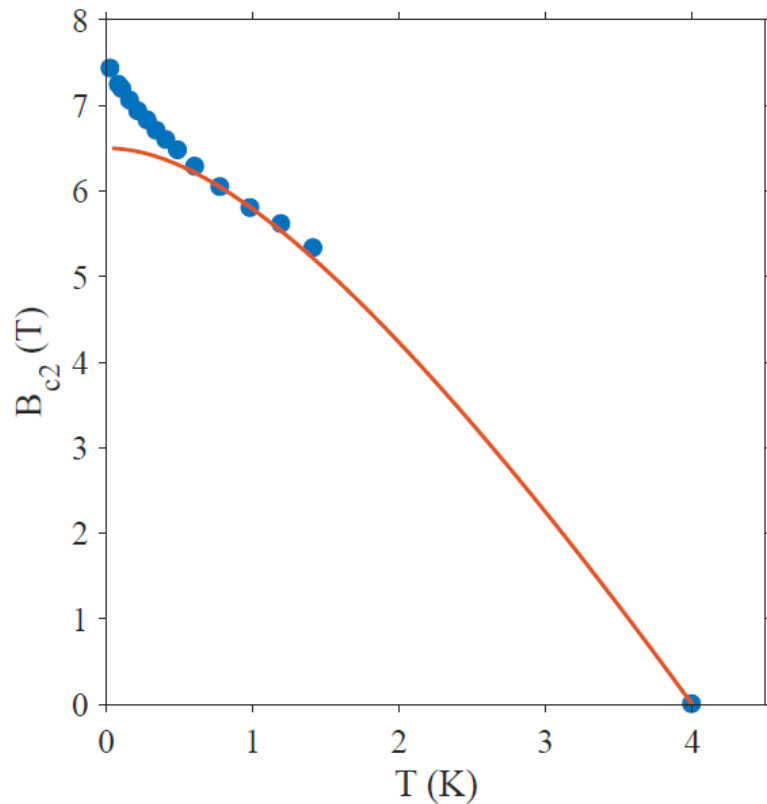


Thermal fluctuations of vortex glass

$$1 - \frac{B_{c2}(T)}{B_{c2}(0)} \sim T$$

Universality

MoGe film : 3 nm thick $R_{sq} = 700 \Omega$ $T_c = 4 K$



Thank you!