Magnetic field enhancement of superconductivity in disordered D-wave superconductors

B.Spivak

University of Washington

with S. Kivelson, C. Laumann and C. Baldwin

order parameter in pure superconductors

$$\Delta(\mathbf{r},\mathbf{r}') = \int \Delta(\mathbf{k}) e^{i\mathbf{k}(\vec{\mathbf{r}}-\mathbf{r}')} d\mathbf{k}$$

order parameter in S-wave superconductors:

$$\Delta(\mathbf{k}) = const$$

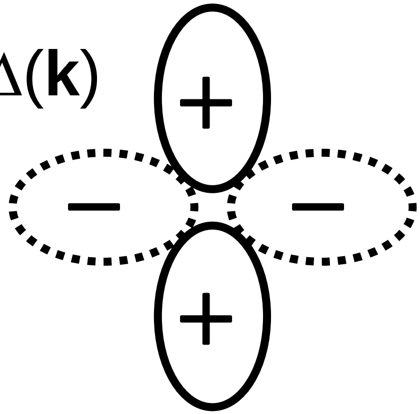
A minimal lattice model describing S-wave superconductivity

$$E = -j\sum_{ij} e^{i\left(\varphi_i - \varphi_j\right)} + c.c., \qquad j > 0$$

φ_i is a phase of order prameter

D-wave order parameter in pure in pure superconductors

$\Delta(\mathbf{k})$



$$\Delta(\mathbf{r},\mathbf{r}') = \int \Delta(\mathbf{k}) e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')} d\mathbf{k}$$

$$\Delta(\mathbf{r} = \mathbf{r}') = 0$$

an assumption: e-e interaction in D-channel is attractive and in S-channel is repulsive

electron scattering destroys D-wave superconductivity when the electron mean free path $l \sim \xi$ becomes of order of the superconducting coherence length.

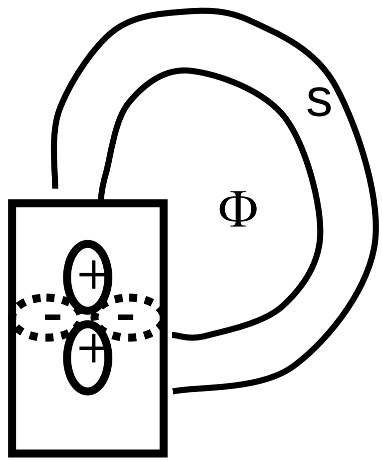
A minimal lattice model describing superconductivity:

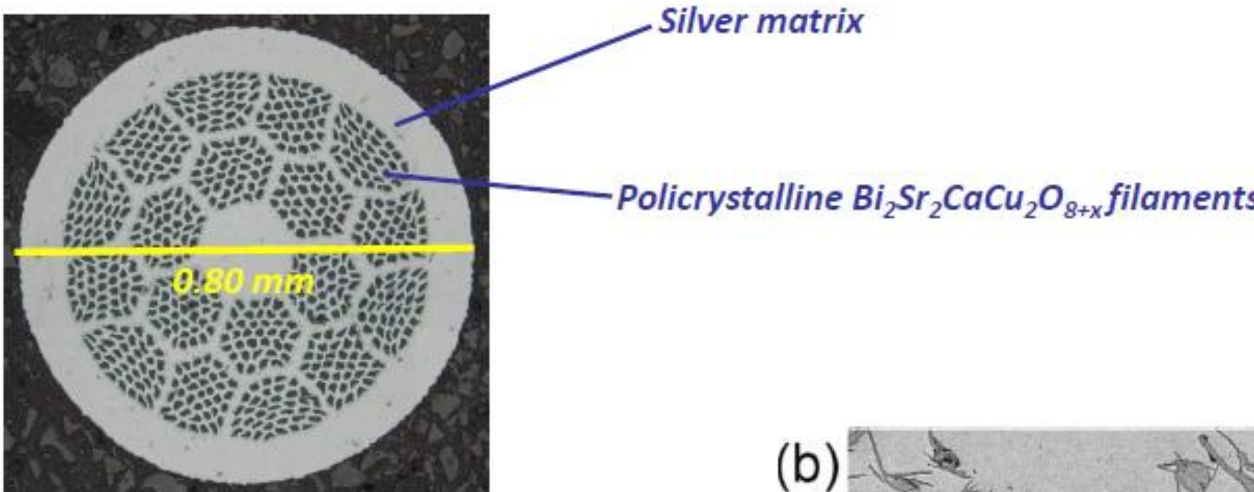
$$E = -j\sum_{ij} e^{i(\varphi_i - \varphi_j)} + c.c., \qquad j > 0$$

φ_i is a phase of order prameter

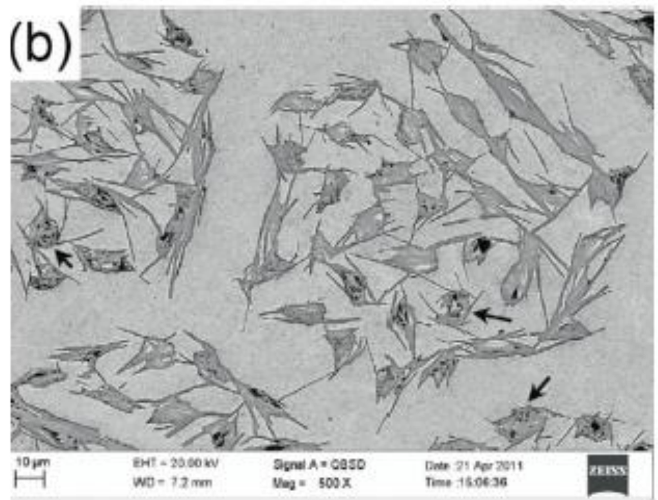
“corner SQUID” experiment demonstrates d-wave symmetry of the order parameter

J.R. Kirtley at all

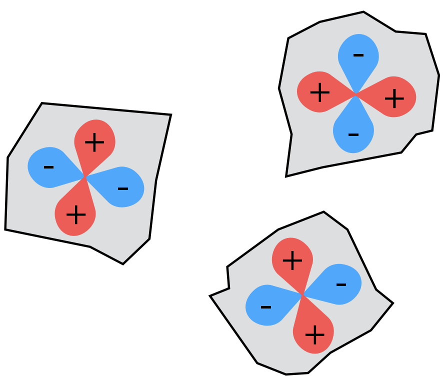




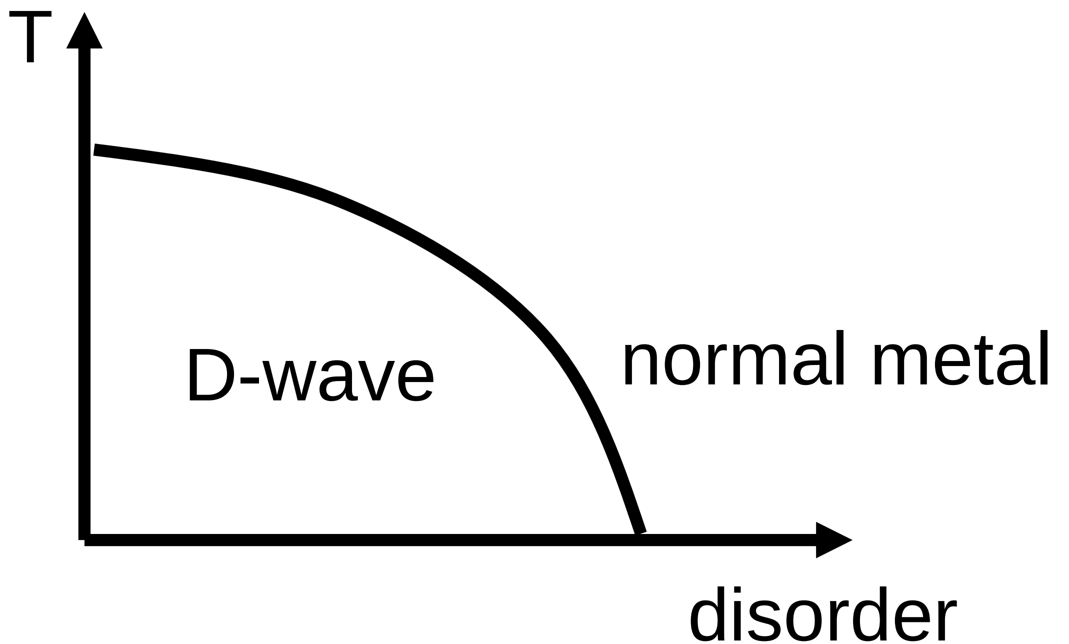
Cross section of a multifilamentary $Bi_2Sr_2CaCu_2O_{8+x}$ wire



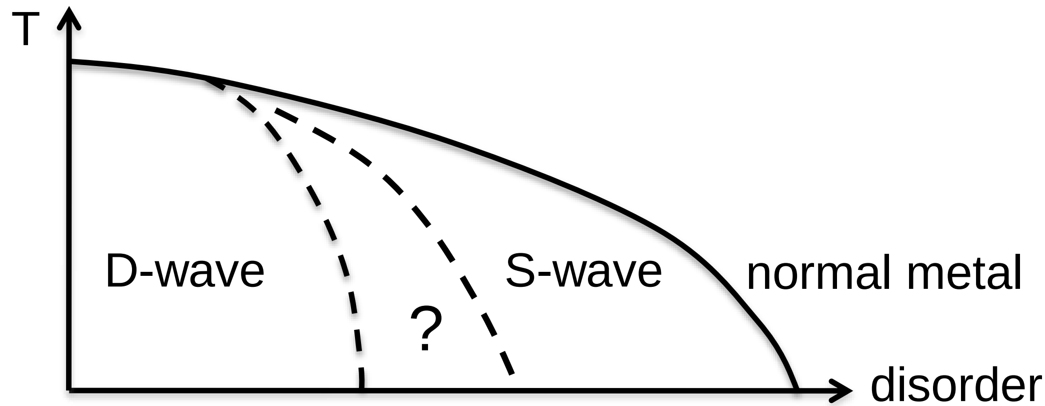
Magnification of the filament bundle

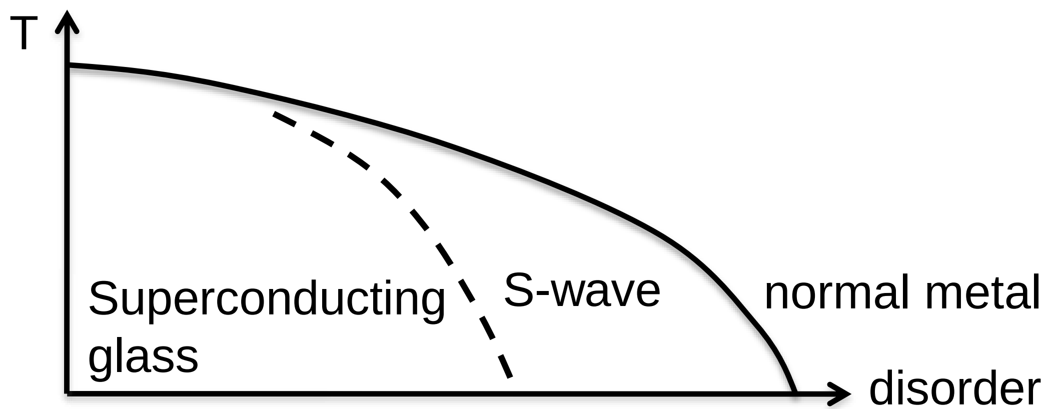


“conventional” phase diagram for D-wave superconductors



phase diagrams of disordered D-wave superconductors





at large disorder the order parameter has global S-wave symmetry regardless of its symmetry in pure state

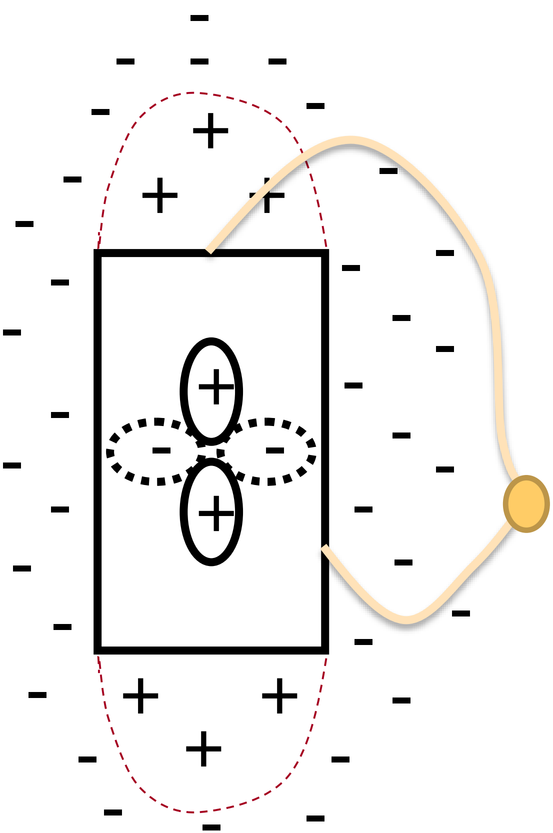
possible definitions of the global S-wave symmetry in bulk disordered samples :

1. corner SQUID experiment shows global s-wave symmetry of the order parameter

2. the quantity $< F_s(\mathbf{r})> = <F(\mathbf{r}=\mathbf{r}')>$ is nonzero. (The brackets stand for averaging over realizations of random potential.)

3. the system has s-wave global symmetry if $P_+ - P_- > (<) 0$. P_+ and P_- are volume fractions where $F(\mathbf{r}=\mathbf{r}') = F_s(\mathbf{r})$ has positive or negative sign, respectively.

d-wave superconducting puddle embedded into disordered normal metal.
outside the puddle s-wave component of the order parameter is generated. Because of the Anderson theorem only this component survives on distances larger than elastic mean free path l



+ and – indicate signs of the s-components of the anomalous Green function $F_s(\mathbf{r},\mathbf{r})$

Far from the grain the sign is determined by one number

$$\eta = sign\left(\oint_S \Delta_s(\vec{r})\, d\vec{r}\right)$$

in diffusive metal s-component of the anomalous Green function $F_s(r)=F(r,r)$ is described by the Usadel equation

$$D_{tr}\frac{d^2\theta(\varepsilon,\vec{r})}{d^2\vec{r}}+i\varepsilon\sin\theta(\varepsilon,\vec{r})=0;\quad F_s(\vec{r},\varepsilon)=-i\sin\theta(\varepsilon,\vec{r})$$

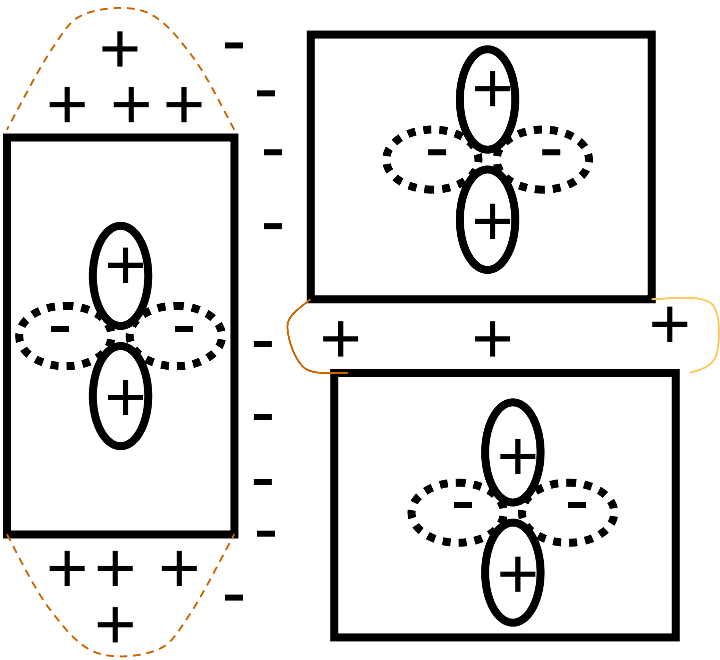
$$F_s(\vec{r})=\int F_s(\vec{r},\varepsilon)d\,\varepsilon$$

D_{tr} *is the electron diffusion coefficient in the normal metal*

boundary conditions at D-N boundary are after Tanaka et. al.

$$F_s(r)\propto A\frac{1}{r^D}\exp\left[-\frac{r}{L_T}\right];\qquad L_T=\sqrt{\frac{D_{tr}}{T}}$$

the sign of the coefficient A is determined by the shape of the puddle if puddle concentration is big the order parameter has global d-wave symmetry, while the s-component of the order parameter has random sample specific sign

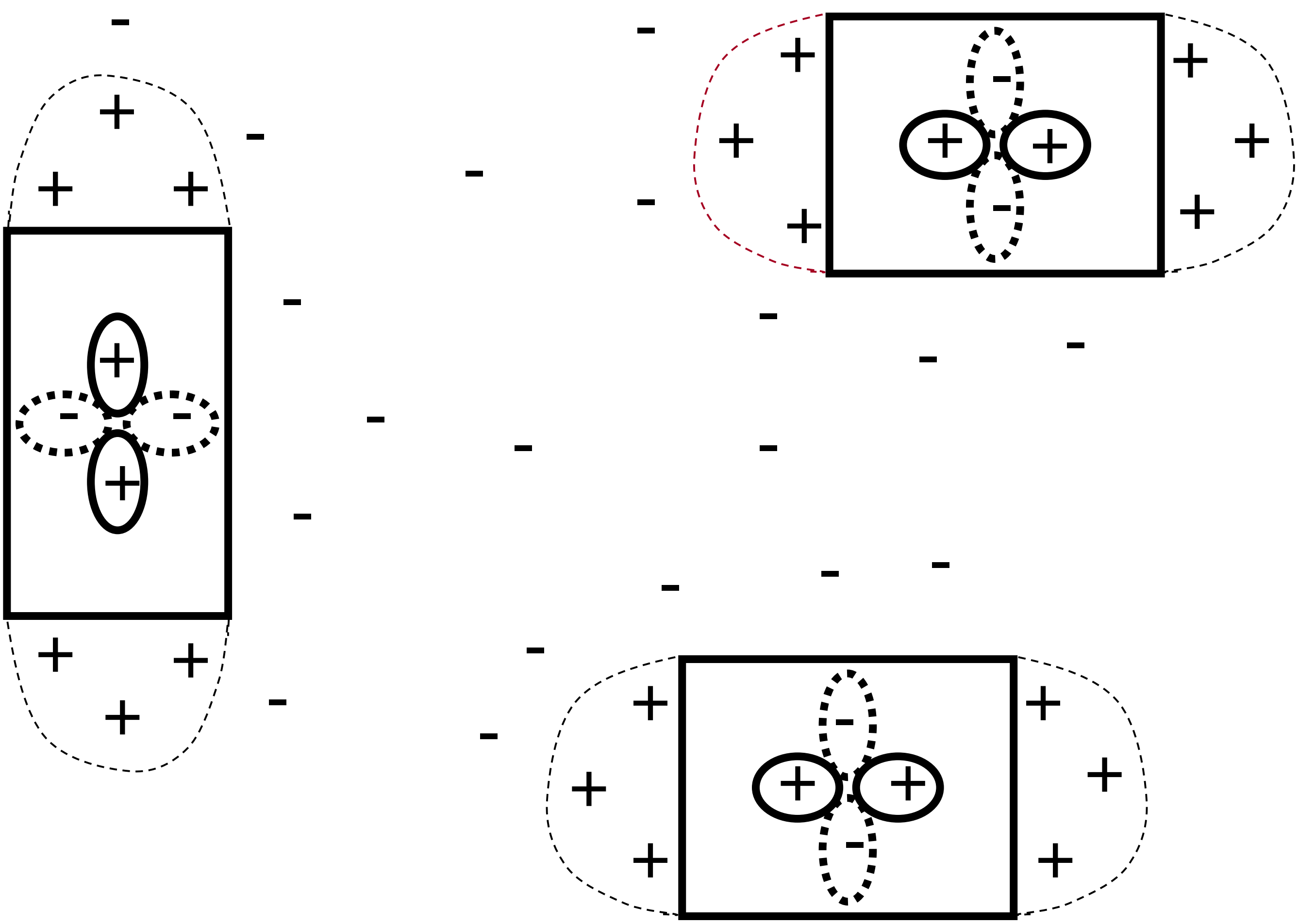


Effective mean field energy

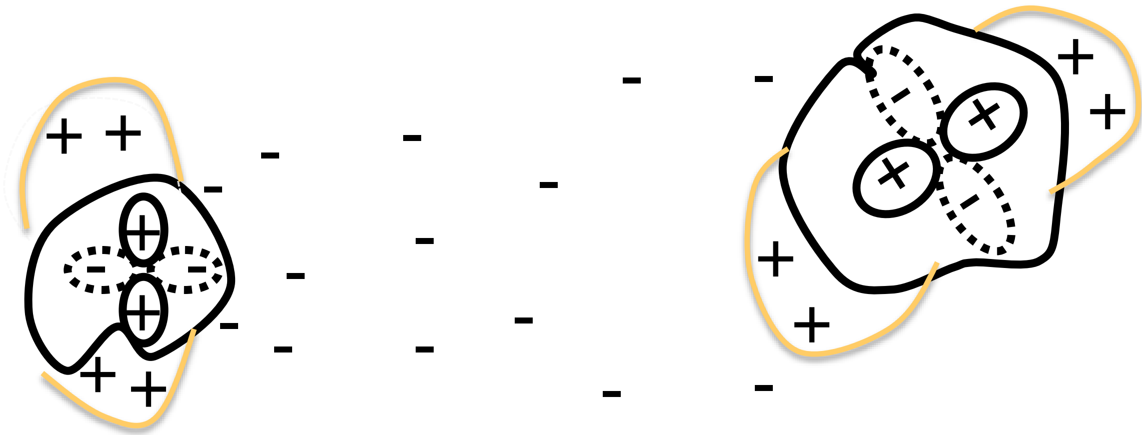
$$E = -\sum_{ij} j_{ij}^{(d)} e^{i(\varphi_i - \varphi_j)} + c.c.$$

$J_{ij}^{(d)} > 0$ is the Joshepson coupling energy between D-wave components

if the concentration of superconducting puddles is small
the order parameter has s-wave global symmetry, while
the d-wave component has random sample specific sign



more realistic picture of superconducting puddles embedded into a metal



effective energy of the system is equivalent to Mattis model :

$$E = -\sum_{ij} j_{ij}^{(s)} \eta_j \eta_i e^{i(\varphi_i - \varphi_j)} + c.c.; \quad \eta_i = \pm 1 \; \textit{are random numbers.}$$

$$\textit{in the ground state} \; e^{i\varphi_i} = \eta_i$$

$J_{ij}^{(s)} > 0$ is the Joshepson coupling energy between S-wave components sufficiently disordered D-wave superconductors behave as S-wave superconductors with respect to all interference experiments

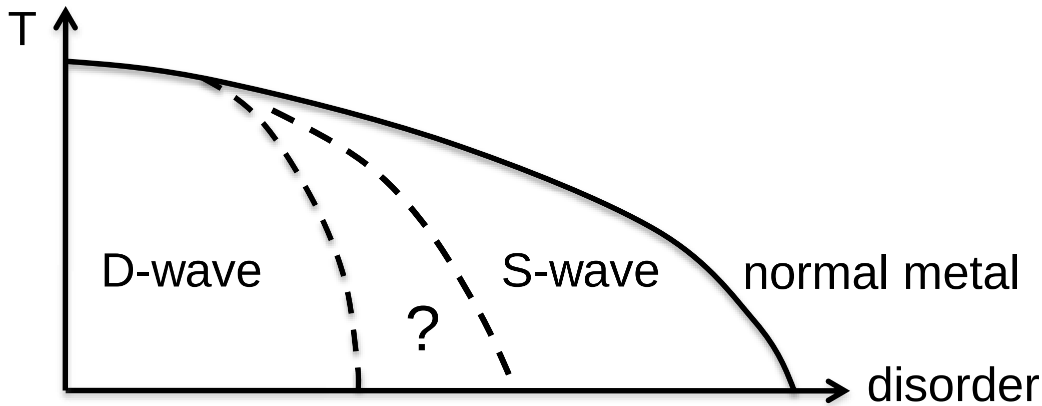
an effective energy at intermediate concentration of superconducting puddles

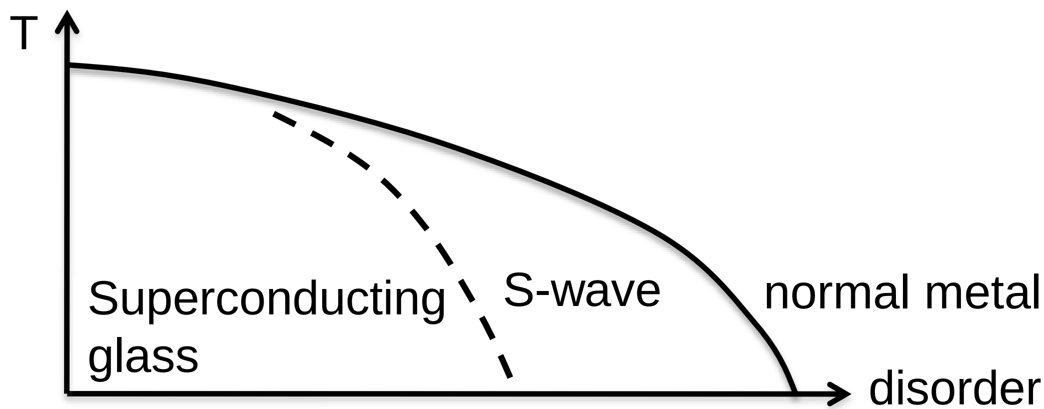
$$E = -\sum_{ij}\left[j_{ij}^{(s)}\eta_j\eta_i + j_{ij}^{(d)}\right]e^{i\left(\varphi_i - \varphi_j\right)} + c.c.$$

$$\eta_i = \pm 1 \; are \; random \; numbers$$

There is a superconducting glass phase when $J^{(s)} \sim J^{(d)}$

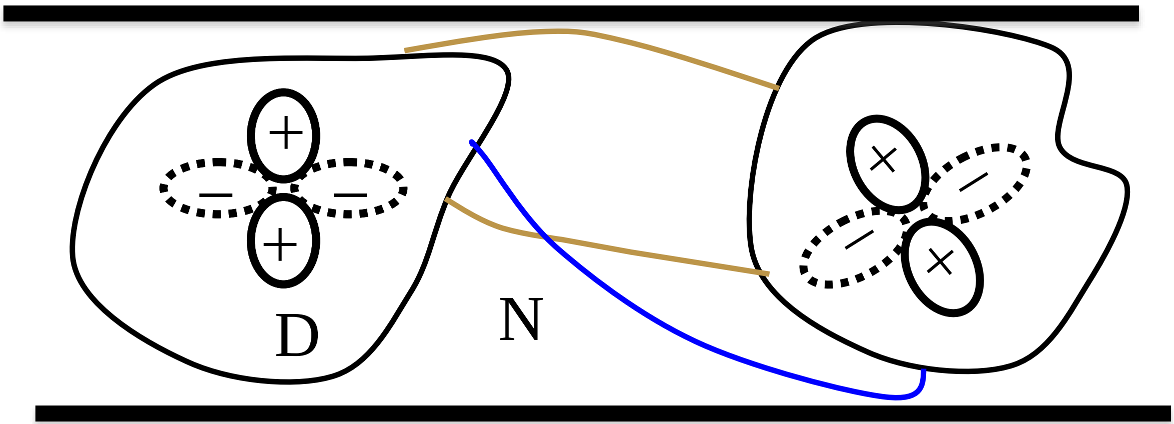
phase diagrams of disordered D-wave superconductors





at large disorder the order parameter has global S-wave symmetry regardless of its symmetry in pure state

small magnetic field enhances superconductivity of disordered d-wave superconductors.
an example: quasi-one-dimensional wire



In the presence of the magnetic field acquire complex phases:

$$\boxed{J_{ij}(\mathbf{H}) = \pm e^{i\zeta_{ij}} \left| A_{ij} - B_{ij} e^{i\chi_{ij}} \right| I_{ij}.} \quad \zeta_{ij} = \mathbf{A}(\mathbf{r}) \cdot \mathbf{r}_{ij}$$

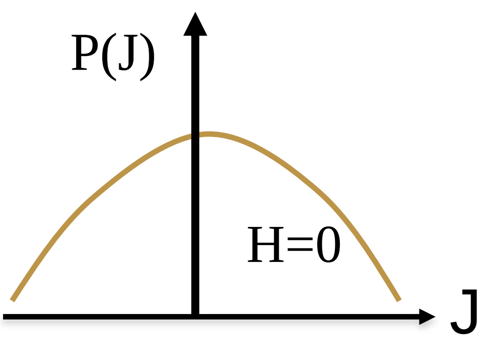
A_{ij} and B_{ij} are appropriately normalized sums of positive and negative diffusive amplitudes. They are random positive quantities.

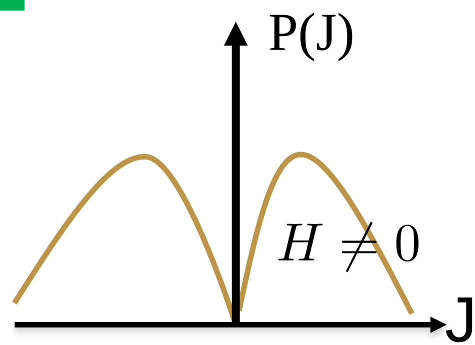
corrections to the superfluid stiffness is positive and non-analytical in the magnetic field H.

$$\langle N_s(H)\rangle = \lim_{L\to\infty}\left\langle L\left(\sum\frac{1}{|J_{ij}|}\right)^{-1}\right\rangle = r_G\left[\int\frac{p(|J|)}{|J|}d|J|\right]^{-1}$$

at H=0 the probability density $p(|J_{ij}|=0)$ is finite.
consequently the integral diverges and the superfluid stiffness N_s is 0. At finite value of H $p(|J_{ij}|=0)=0$

$$\boxed{\langle N_s(H)\rangle \sim \frac{N_s^{(0)}}{|\log(\phi^2)|}.}\quad \phi=(HS/\Phi_0)$$





magnetic field enhancement of superconductivity in the Mattice phase, 2D and 3D cases:

if the Josephson coupling decays exponentially with the inter-grain distance, one can use the percolation theory to calculate both the global superfluid stiffeness and the critical temperature

$$\boxed{J_{ij}(\mathbf{H}) = \left|A_{ij} - B_{ij} e^{i\chi_{ij}}\right| J_0 e^{-r_{ij}/L_T}.}$$

$$J_{ij} = J_0 \exp(-\xi_{ij}), \qquad \xi_{ij} = \xi_{ij}^{(0)} + \delta\xi_{ij}$$

$$\xi_{ij}^{(0)} = r_{ij}/L_T, \quad \delta\xi_{ij} = -\ln\left|A_{ij} - B_{ij} e^{i\chi_{ij}}\right|.$$

A_{ij} and B_{ij} are appropriately normalized sums of positive and negative diffusive amplitudes in the absence of the magnetic field.

calculation of the H-dependence of the superfluid stiffness is similar to a calculation of the magnetoresistance in hopping conductivity regime

1) $\delta\xi_{ij} = 0$ $\langle N_s^{(0)} \rangle = J_0 r_G^{2-d} \left(\frac{L_T}{r_c^{(0)}} \right)^{\nu} e^{-\frac{r_c^{(0)}}{L_T}}$

$$r_c^{(0)} = L_T \xi_c^{(0)}$$

2) A prescription of the perturbation theory of the percolation theory: one has to average the log

$$\delta\xi_c = \langle \delta\xi_{ij} \rangle.$$

$$\begin{aligned} \delta\xi_c(H) &= -\left\langle \ln \left| A_{ij} - B_{ij} e^{i\chi_{ij}} \right| \right\rangle \\ &= -\int_{-\pi\phi}^{\pi\phi} \frac{\mathrm{d}\chi}{2\pi\phi} \int_0^1 \mathrm{d}A \ln \left| A - (1-A) e^{i\chi} \right| \\ &\sim 1 - \frac{\pi^2}{8} |\phi| \end{aligned}$$

$$\phi = (HS/\Phi_0)$$

correction to the superfluid density is positive and non-analytical (linear) in the magnetic field H.

$$\phi > e^{-\xi_c}$$

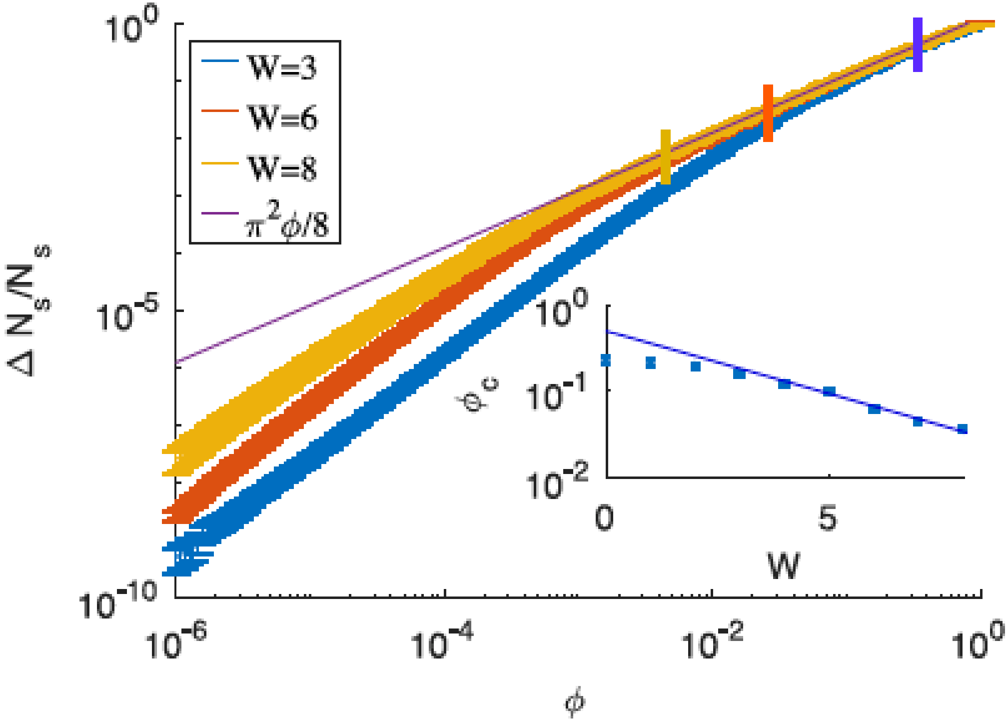
$$\boxed{\frac{\langle N_s(H)\rangle - \langle N_s(0)\rangle}{\langle N_s(0)\rangle} \sim \frac{\pi^2}{8}\frac{|H|S}{\Phi_0}}$$

$$\phi < e^{-\xi_c}$$

$$\boxed{\frac{\langle N_s(H)\rangle - \langle N_s(0)\rangle}{\langle N_s(0)\rangle} \sim c(\frac{|H|S}{\Phi_0})^2}$$

$$c \sim e^{\xi_c} \gg 1$$

numerical simulations of the magnetic field dependence of the superfluid density



magnetic field enhancement of the critical temperature is non-analytical in the magnetic field

$$T_c(H) < \frac{\hbar}{2e}|J_{ij}|$$

$$\xi_c \equiv r_c/L_T$$

$$T_c = \frac{\hbar}{2e}J_0 \exp\left(-r_c/L_{T_c}\right).$$

$$\boxed{\frac{T_c(H) - T_c(0)}{T_c(0)} \sim \frac{\pi^2}{4}\frac{L_{T_c(0)}}{r_c^{(0)}}\frac{|HS|}{\Phi_0}.}$$

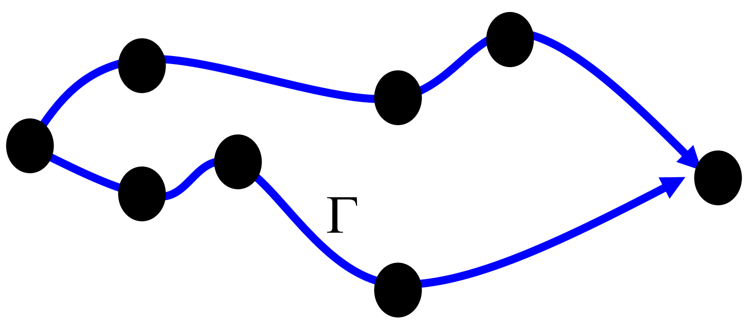
the magnetic field enhancement of superconductivity in the superconducting glass phase

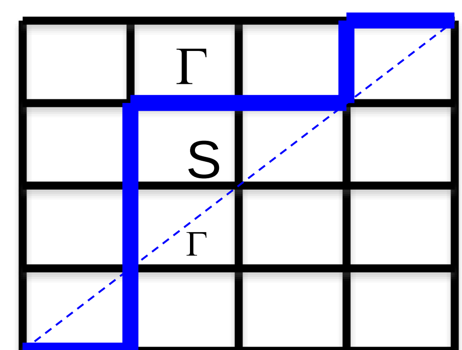
at high-temperature the superconducting phase correlation function

$$A_{kl} = \langle e^{i(\theta_k - \theta_l)} \rangle = \mathrm{Tr}\left[e^{i(\theta_k - \theta_l)} \frac{e^{-\beta H}}{Z} \right],$$

can be expressed as a sum of directed paths amplitudes

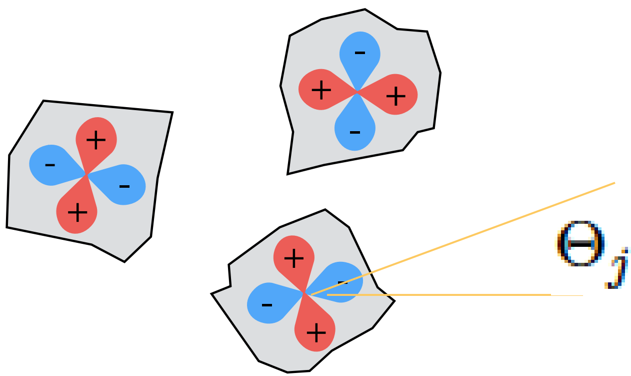
$$\boxed{A_{kl} = \sum_{\Gamma} A_{\Gamma}, \quad A_{\Gamma} \equiv \prod_{\langle ij \rangle \in \Gamma} \left(\frac{\pi \hbar \beta}{2e} J_{ij} \right).}$$





$$\boxed{J_{ij} = J_0\, e^{i\zeta_{ij}} \cos 2(\Theta_i - \Theta_j),}$$

Θ_i are the orientation of the nodes of the order parameter on superconducting grains



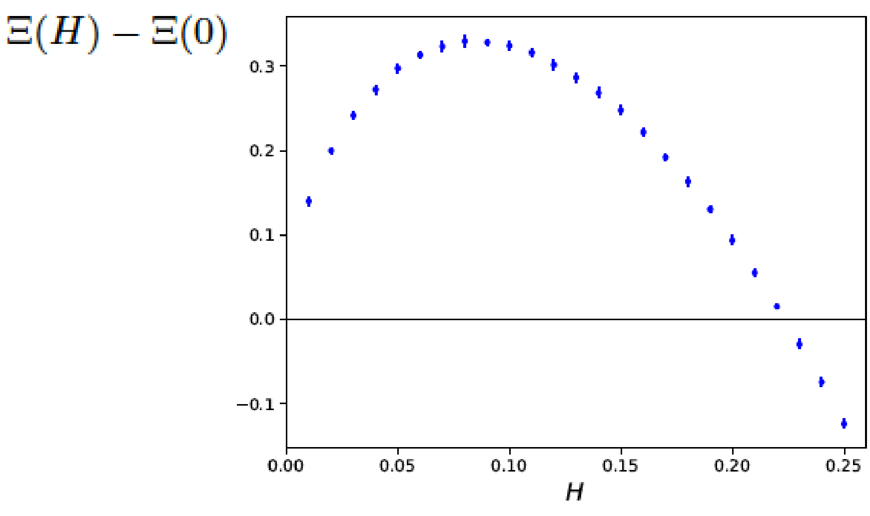
does the superconducting phase correlation function have random sign?

Magnetic field corrections to the localization radius are positive and non-analytic in H

$$\langle \ln |A_{kl}| \rangle \sim -r/\Xi(H).$$

$$\boxed{\frac{\Xi(H)-\Xi(0)}{\Xi(0)} \sim \left(\frac{\Xi(0)^2\,|H|}{\Phi_0}\right)^{\alpha}}$$

$$\alpha = 0.59 \pm 0.03. \qquad \Xi(0) = \left(\ln \frac{2e}{\pi\hbar\beta J_0}\right)^{-1}$$



Conclusion:

in between of D-wave superconductor (or superconducting glass) and normal metal there is superconducting phase with S-wave symmetry

small magnetic field enhances superconductivity of disordered D-wave superconductors