Higgs mode in superfluids and superconductors

Nandini Trivedi,
The Ohio State University

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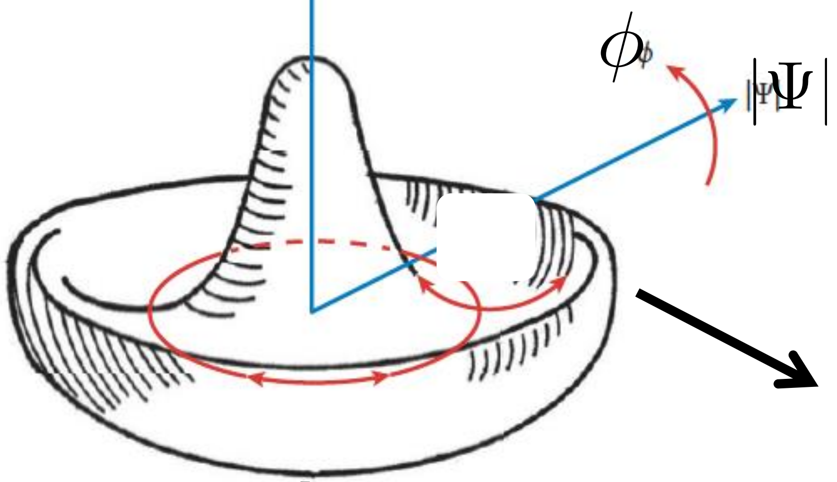
NSF-DMR- 1309461

Spontaneously broken continuous symmetry

$$\langle a \rangle \equiv \Psi = |\Psi| e^{i\phi}$$



Particle physics

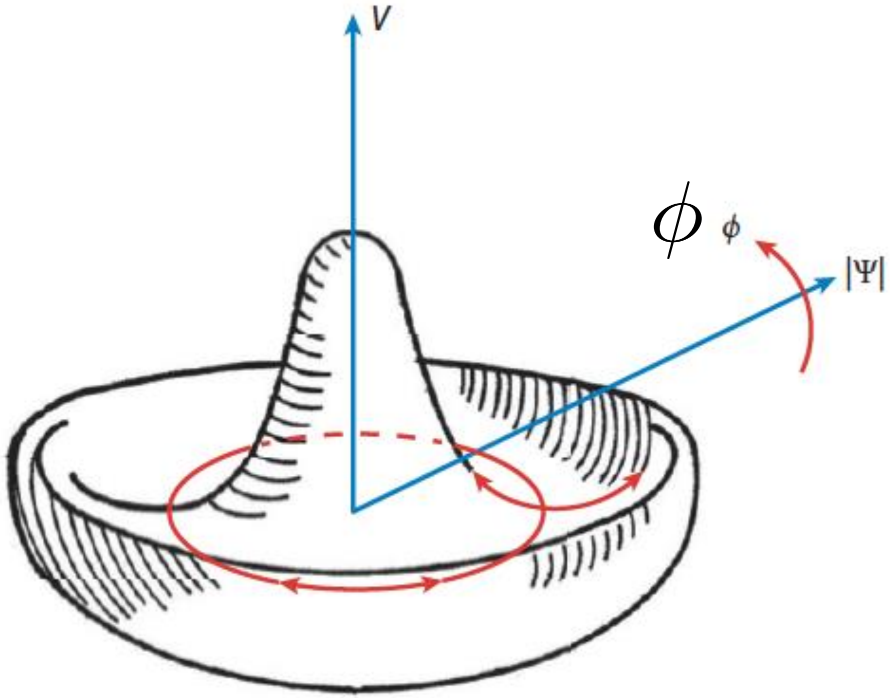
Superfluid

Superconducto
r

Gapless azimuthal phase mode: Goldstone mode

Gapped radial amplitude mode: Higgs mode

$$\langle a \rangle \equiv \Psi = |\Psi| e^{i\phi}$$



ϕ $|\Psi|$

Prevailing notions:

- Pure amplitude mode can only be observed in a Lorentz invariant system
- Pure amplitude mode can only be observed in a clean non-disordered system
- Is CDW necessarily required to observe a Higgs mode?

Are these true?

Further challenges of observing the Higgs mode in a superconductor:

Within BCS theory,

Higgs energy scale ~ 2Δ the pair breaking scale

hence Higgs mode is heavily damped

Previous works:

Nonrelativistic Dynamics of the Amplitude (Higgs) Mode in Superconductors
T. Cea, C. Castellani, G. Seibold, and L. Benfatto PRL 115, 157002 (2015)

Nonlinear optical effects and third-harmonic generation in superconductors:
Cooper pairs versus Higgs mode contribution, T. Cea, C. Castellani, and L. Benfatt
PRB 93, 180507 (2016)

Visibility of the Amplitude (Higgs) Mode in Condensed Matter,
Podolsky, Auerbach, Arovas, PRB 84, 174522 (2011)

Fate of the Higgs Mode Near Quantum Criticality
Snir Gazit, Daniel Podolsky, and Assa Auerbach, PRL 110, 140401 (2013)

The 'Higgs' Amplitude Mode at the Two-Dimensional Superfluid-Mott Insulator
Transition, Endres, Fukuhara, Pekker, Cheneau, Schauß, Gross, Demler,
Kuhr, and Bloch, Nature 487, 454 (2012).

Higgs in Bose Hubbard model:

Marco Di Liberto   Alessio Recati

Iacopo Carusotto   Chiara
Menotti

University of Trento

PRL **120,** 073201 (2018)

Higgs in a Superconductor: Fermi Hubbard Model

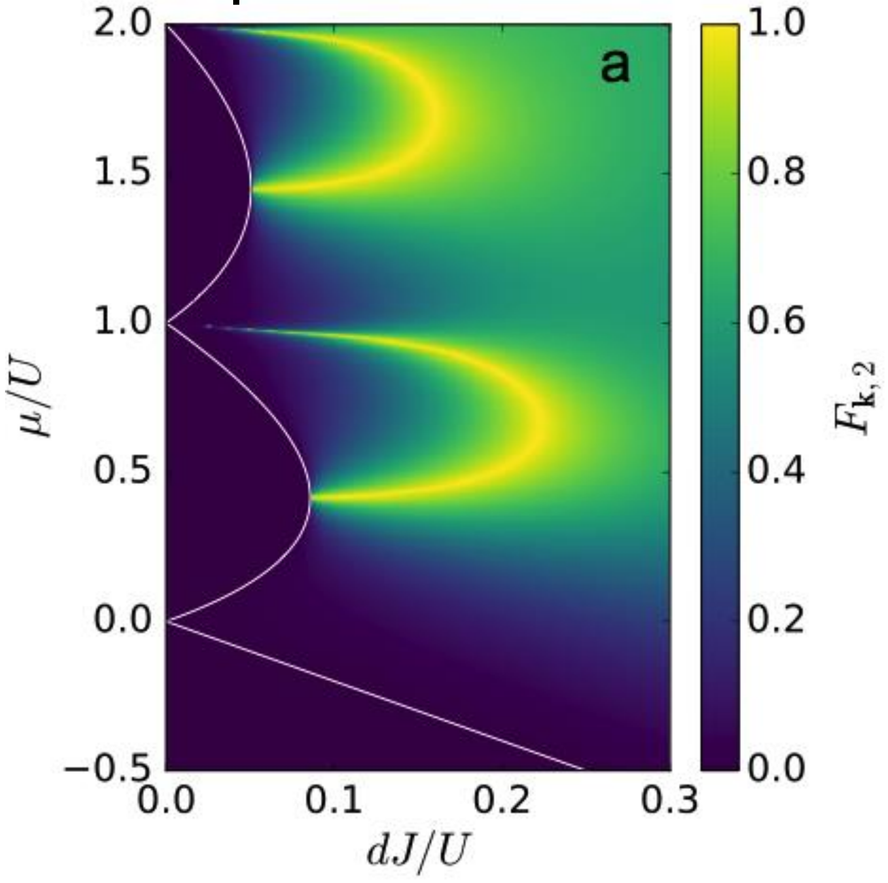
Abhishek Samanta   Amulya Ratnakar   Rajdeep Sensarma

Tata Institute of Fundamental Research

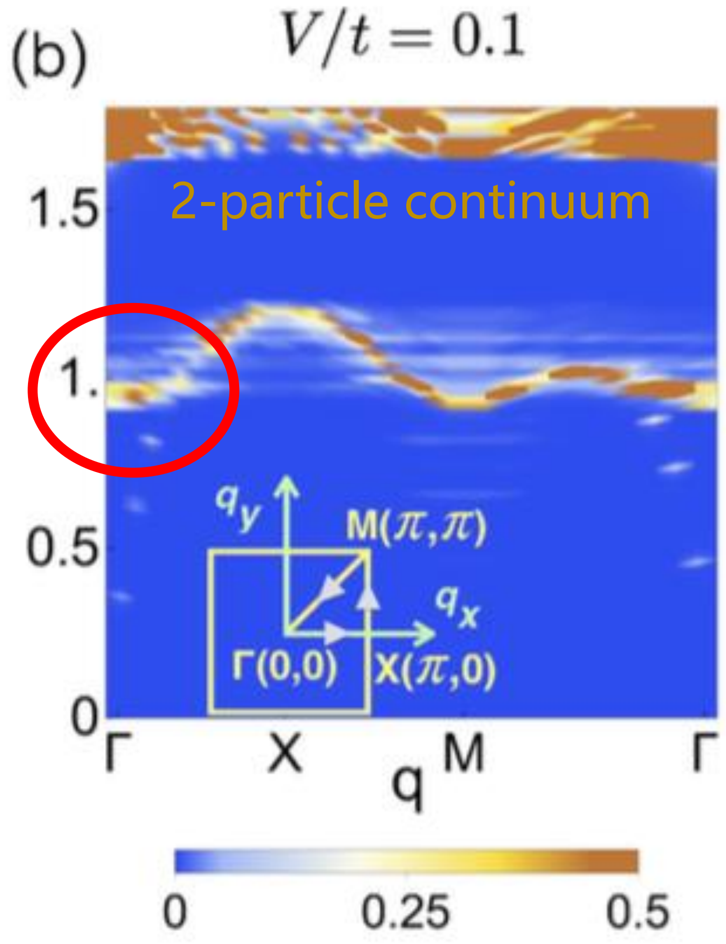
arXiv: 1806.01288

Key Results:

Bose Hubbard Model: emergent particle-hole symmetry away from Lorentz invariant points



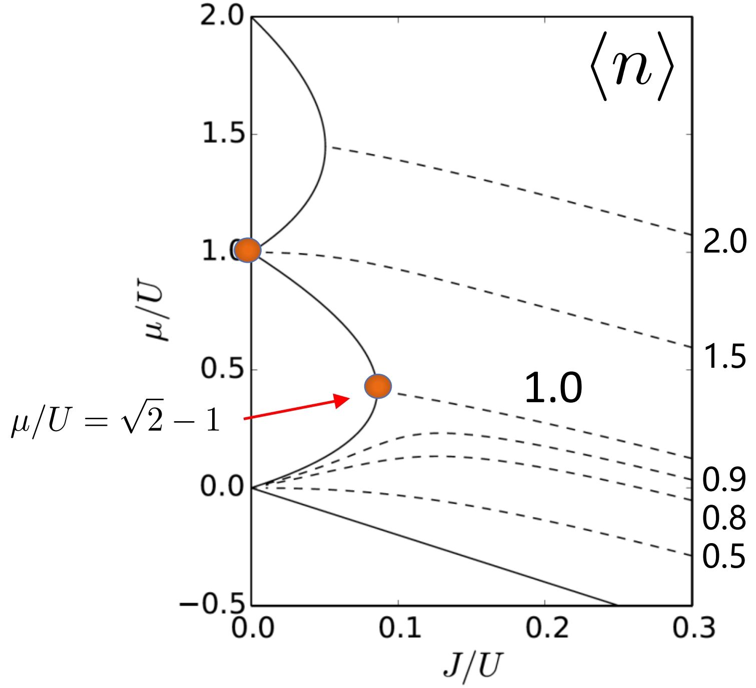
Fermi Hubbard Model:
Higgs mode revealed by disorder!



Does existence of Higgs modes require Lorentz invariance?

At Lorentz invariant point → O(2) relativistic field theory: decoupling of amplitude and phase degrees

What is the fate of the Higgs mode away from the Lorentz invariant point?



Bose Hubbard Model

$$H = -J \sum_{\langle i,j \rangle} \left(a_i^\dagger a_j + \text{H.c.} \right) + \frac{U}{2} \sum_i n_i (n_i - 1) - \mu \sum_i n_i$$

Gapped modes and spectral function

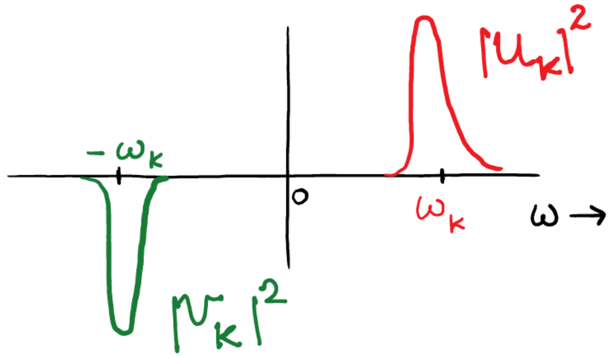
Sum rule

$$\int_{-\infty}^{\infty} A(k,\omega) = 1 \qquad \forall k$$

Boson Spectral function

Bogoliubov Superfluid

$$A_k(\omega) = -\frac{1}{\pi}\,\mathrm{Im}\, G^{ret}_k(\omega)$$



$$A_k(\omega) = |u_k|^2\,\delta(\omega-\omega_k) - |v_k|^2\,\delta(\omega+\omega_k)$$

$$\int_{-\infty}^{\infty} A_k(\omega) = |u_k|^2 - |v_k|^2 = 1$$

sum rule

Spectral function of a superfluid

$$\int_{-\infty}^{\infty} \mathcal{A}(k,\omega) = 1 \qquad \forall k$$

WEAKLY INTERACTING SUPERFLUID

- Bogoliubov theory
- Only one excited mode (Goldstone)
- Spectral weight sum-rule: Goldstone mode is particle dominated

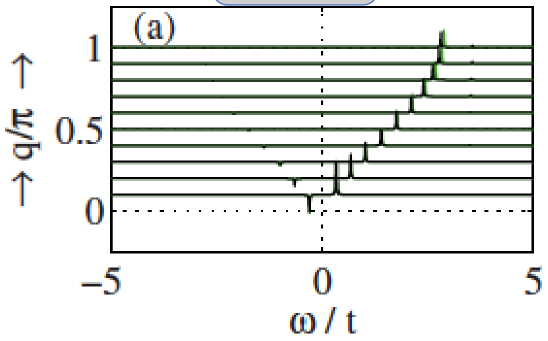
$$|\mathcal{U}_{\mathbf{k}}|^2 - |\mathcal{V}_{\mathbf{k}}|^2 = 1$$

STRONGLY INTERACTING SUPERFLUID

- Several modes required to exhaust the (single-particle) spectral weight

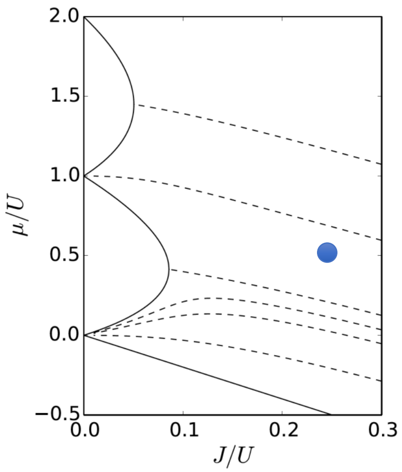
$$\sum_{\lambda} |\mathcal{U}_{\mathbf{k},\lambda}|^2 - |\mathcal{V}_{\mathbf{k},\lambda}|^2 = 1$$

$\mathcal{A}(k,\omega)$

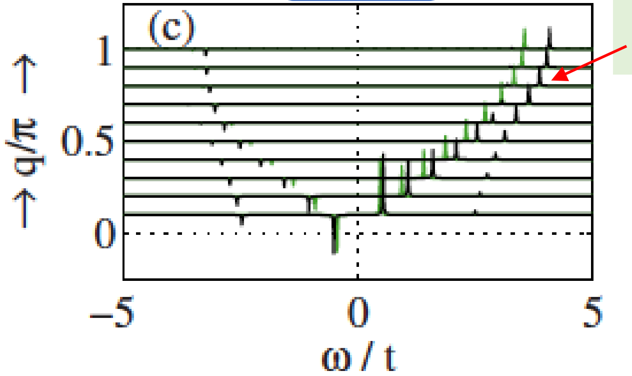


t/U=10

$\mu/U = 0.5$



$\mathcal{A}(k,\omega)$



t/U=0.25

Is the gapped mode a Higgs?

Menotti, Trivedi PRB **77**, 235120 (2008)

Excitations in the Gutzwiller ansatz

Krutitsky & Navez, PRA 2011

$$|\psi\rangle = \bigotimes_i \sum_n c_{i,n}(t)|n\rangle_i$$

Gutzwiller ansatz: product state

$$c_{i,n}(t) = [\bar{c}_n + \delta c_{i,n}(t)]\, e^{-i\omega_0 t}$$

Ground-state coefficients (equilibrium)

Small oscillations

$$\delta c_{i,n}(t) = u_{\mathbf{k},n} e^{i(\mathbf{k}\cdot\mathbf{r}_i - \omega_{\mathbf{k}} t)} + v_{\mathbf{k},n} e^{-i(\mathbf{k}\cdot\mathbf{r}_i - \omega_{\mathbf{k}} t)}$$

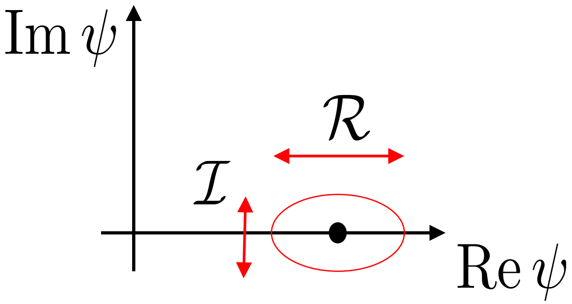
$$L[c, c^*] \equiv i\hbar \sum_{i,n} c^*_{i,n} \partial_t c_{i,n} - \langle H \rangle$$

Linearization of the e.o.m. yields Bogoliubov-like equations

E. Altman and A. Auerbach, PRL 89, 250404 (2002); D. Podolsky, A. Auerbach, and D. P. Arovas, PRB 84, 174522 (2011).
S. Huber, E. Altman, H. Büchler, and G. Blatter, PRB 75, 085106 (2007).

Emergent particle-hole symmetry on yellow contour

$$\delta\psi_{i,\lambda} = \underbrace{\mathcal{U}_{\mathbf{k},\lambda} e^{i(\mathbf{k}\cdot\mathbf{r}_i - \omega_{\mathbf{k},\lambda} t)}}_{\text{Quasi-particle}} + \underbrace{\mathcal{V}_{\mathbf{k},\lambda} e^{-i(\mathbf{k}\cdot\mathbf{r}_i - \omega_{\mathbf{k},\lambda} t)}}_{\text{Quasi-hole}}$$



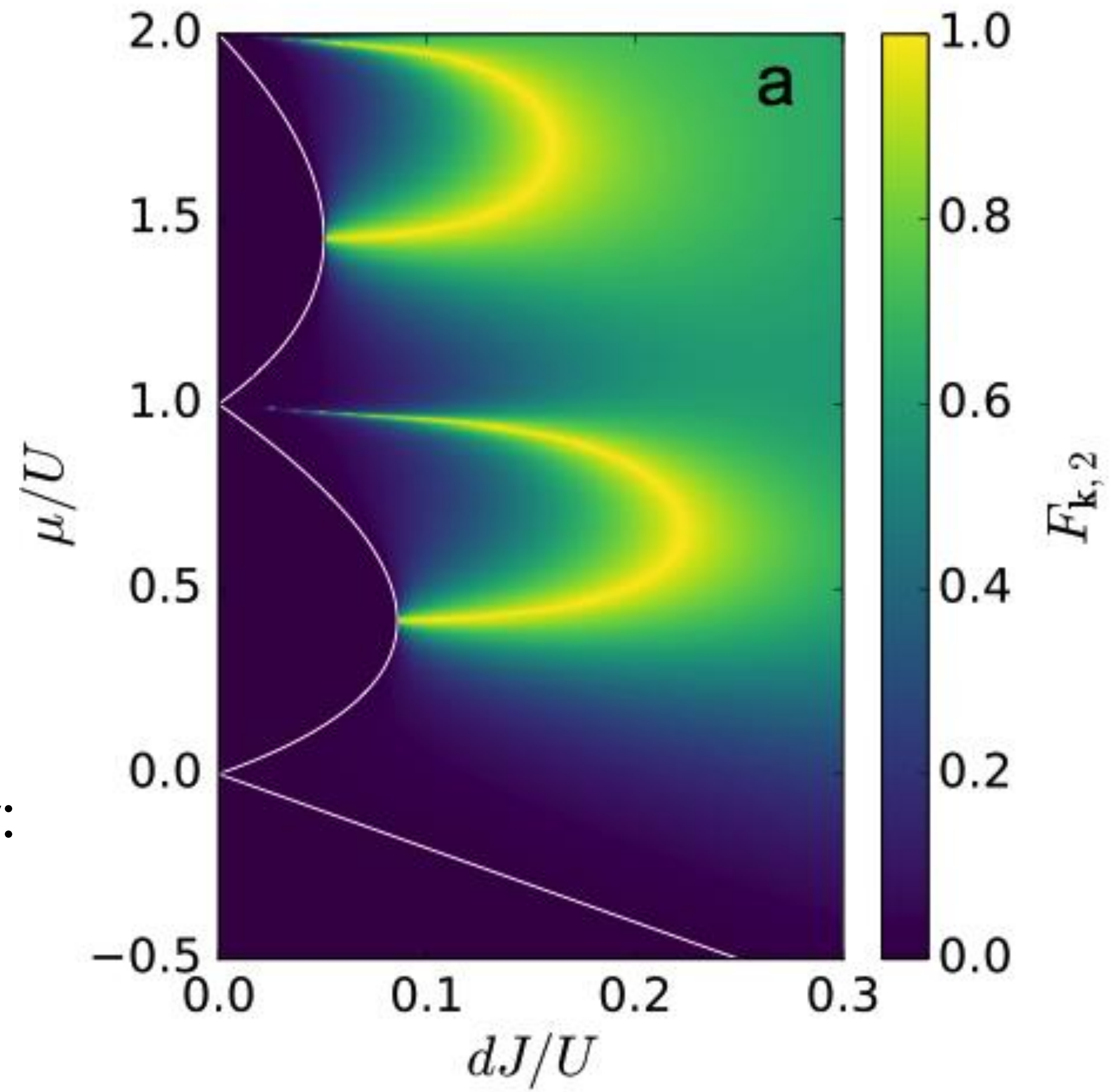
$$\mathcal{R}_{\mathbf{k},\lambda} = (\mathcal{U}_{\mathbf{k},\lambda} + \mathcal{V}_{\mathbf{k},\lambda})$$

$$\mathcal{I}_{\mathbf{k},\lambda} = (\mathcal{U}_{\mathbf{k},\lambda} - \mathcal{V}_{\mathbf{k},\lambda})$$

$$\boxed{F_{\mathbf{k},\lambda} = \frac{\mathcal{R}_{\mathbf{k},\lambda} - \mathcal{I}_{\mathbf{k},\lambda}}{\mathcal{R}_{\mathbf{k},\lambda} + \mathcal{I}_{\mathbf{k},\lambda}}}$$

On yellow contours p-h symmetry:

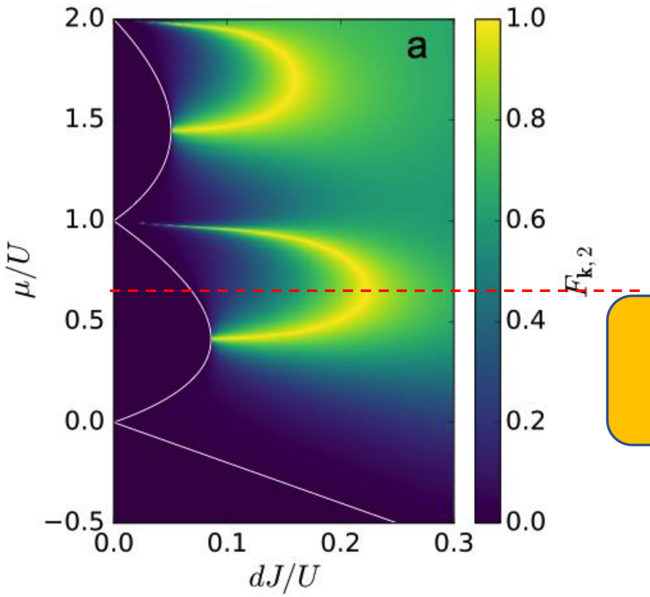
$$\boxed{\mathcal{U}_{\mathbf{k},\lambda} = \mathcal{V}_{\mathbf{k},\lambda}}$$



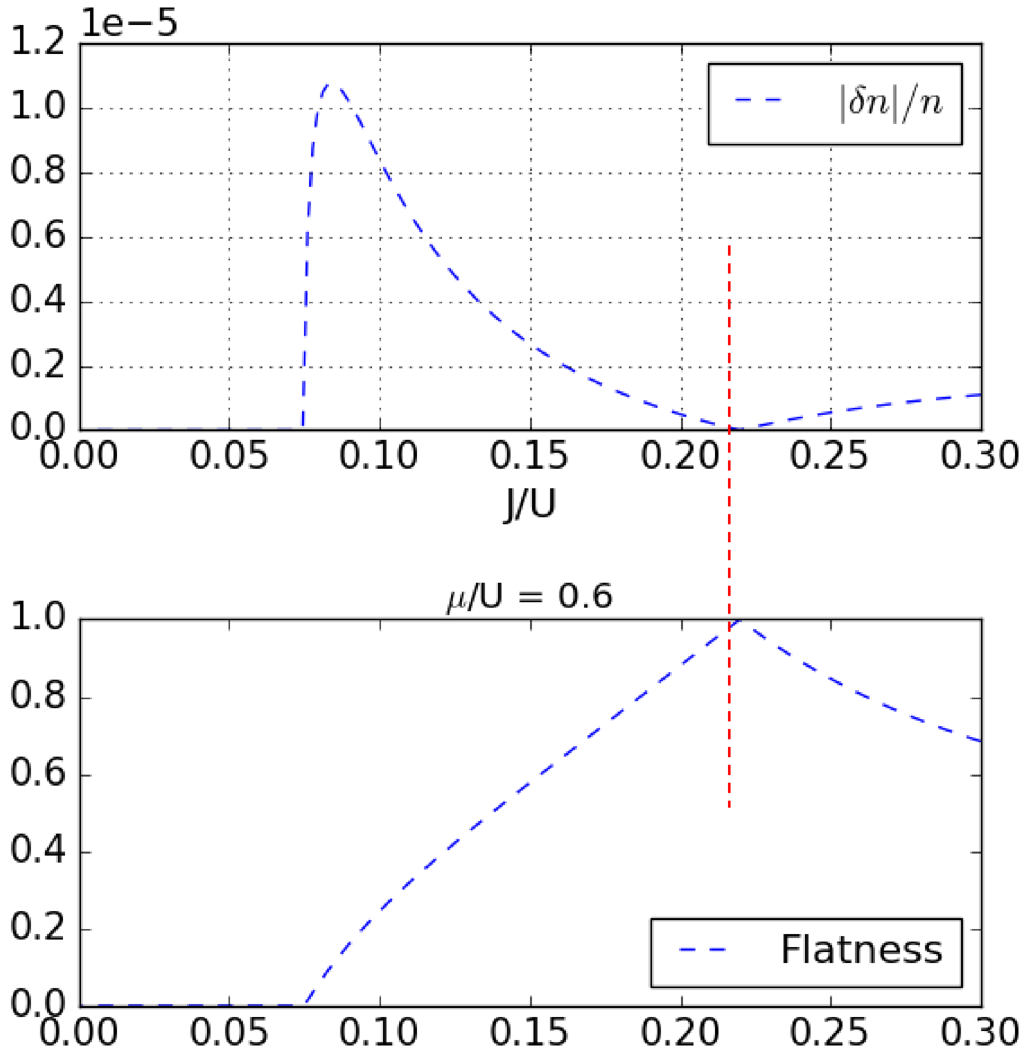
Vanishing density fluctuations on yellow contours

$$\delta n = 0$$

1. Density oscillations vanish exactly: exchange of particles between condensate and normal fraction
2. Higher modes crucial for curving of yellow arc



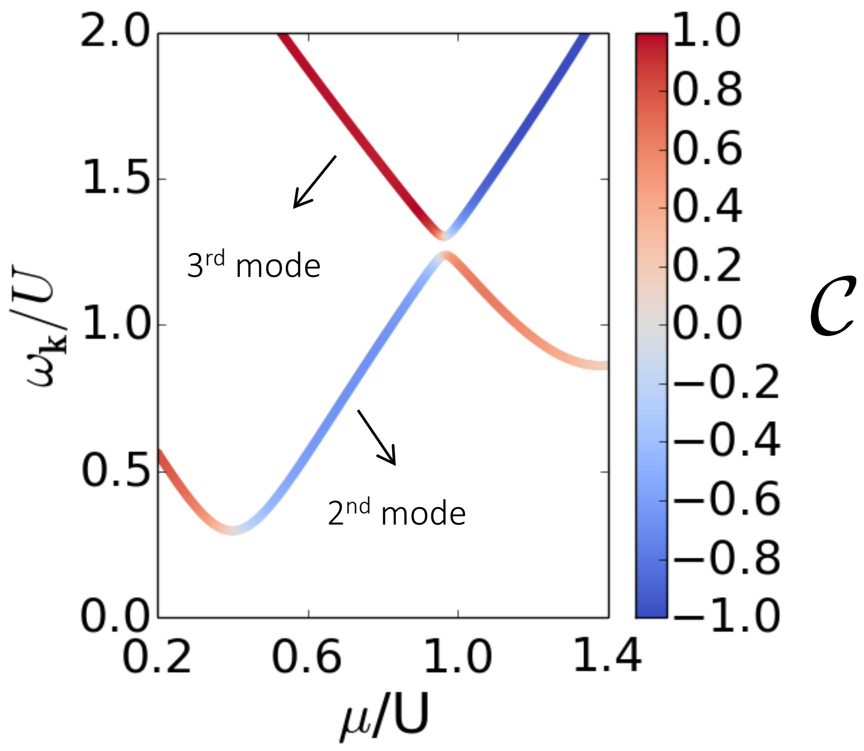
$$\mathcal{U}_{\mathbf{k},\lambda} = \mathcal{V}_{\mathbf{k},\lambda}$$



Emergent p-h symmetry

$$\delta\psi_{i,\lambda} = \underbrace{\mathcal{U}_{\mathbf{k},\lambda} e^{i(\mathbf{k}\cdot\mathbf{r}_i - \omega_{\mathbf{k},\lambda} t)}}_{\text{Quasi-particle}} + \underbrace{\mathcal{V}_{\mathbf{k},\lambda} e^{-i(\mathbf{k}\cdot\mathbf{r}_i - \omega_{\mathbf{k},\lambda} t)}}_{\text{Quasi-hole}}$$

$$\mathcal{C} = \frac{|\mathcal{U}_{\mathbf{k},\lambda}| - |\mathcal{V}_{\mathbf{k},\lambda}|}{|\mathcal{U}_{\mathbf{k},\lambda}| + |\mathcal{V}_{\mathbf{k},\lambda}|}$$



$$\mathcal{U}_{\mathbf{k},\lambda} = \sum_n \sqrt{n+1}\,(\bar{c}_n u^{(\lambda)}_{\mathbf{k},n+1} + \bar{c}_{n+1} v^{(\lambda)}_{\mathbf{k},n})$$

$$\mathcal{V}_{\mathbf{k},\lambda} = \sum_n \sqrt{n+1}\,(\bar{c}_{n+1} u^{(\lambda)}_{\mathbf{k},n} + \bar{c}_n v^{(\lambda)}_{\mathbf{k},n+1})$$

Particle-hole symmetry condition

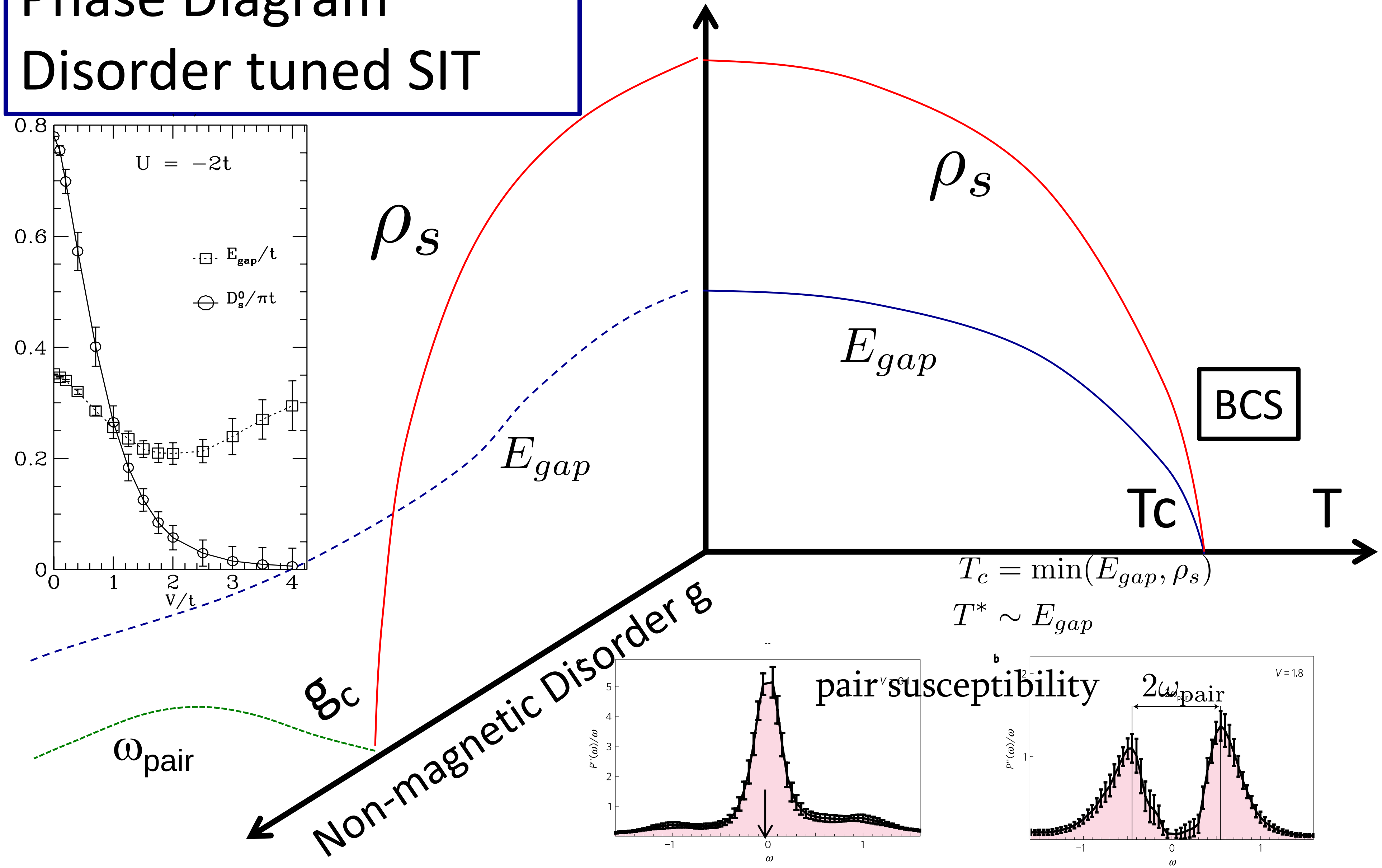
$$|\mathcal{U}_{\mathbf{k},\lambda}| = |\mathcal{V}_{\mathbf{k},\lambda}|$$

Avoided crossing with higher modes yields exchange of particle-hole character

Emergent particle-hole symmetry

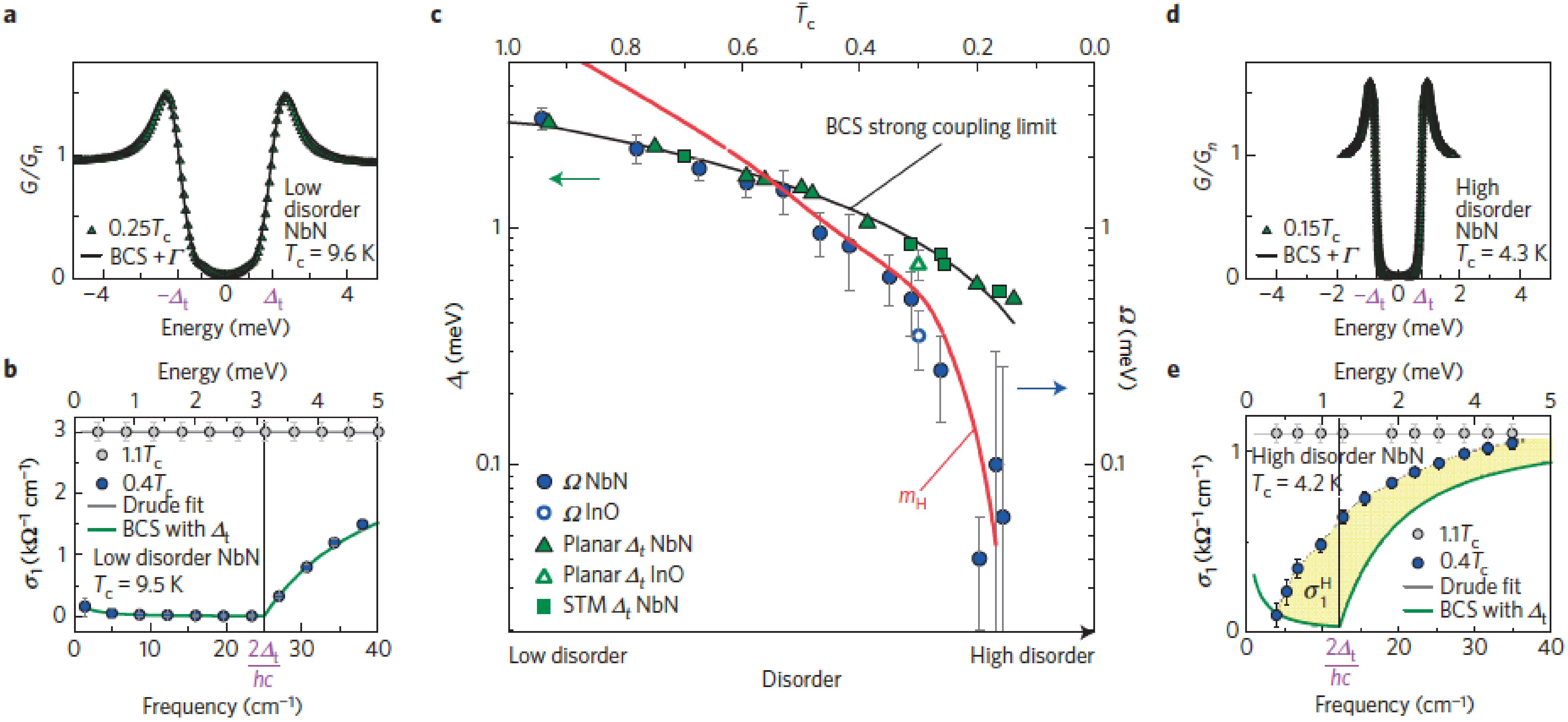
Superconductor

Phase Diagram
Disorder tuned SIT



K. Bouadim, Y. L. Loh, M. Randeria, and N. Trivedi, Nat. Phys., 7, 884 (2011).
A. Ghosal, M. Randeria, and N. Trivedi, PRL 81, 3940 (1998); PRB 65, 014501 (2001).

The Higgs mode in disordered superconductors close to a quantum phase transit



Sherman, Pracht, Gorshunov, Poran, Jesudasan, Chand, Raychaudhuri, Swanson6, Trivedi, Auerbach, Scheer, Frydman, and Dressel, Nat. Phys. 11, 188 (2015)

Amplitude and phase fluctuations

Add spatio-temporal fluctuations around saddle-point value Δ_0

$$\boxed{\Delta(i,\tau) = (\Delta_0(i) + \eta(i,\tau))e^{i\theta(i,\tau)}}$$

Saddle point value

amplitude fluctuation

phase fluctuation

The Gaussian action in amplitude and phase fluctuations around saddle point :

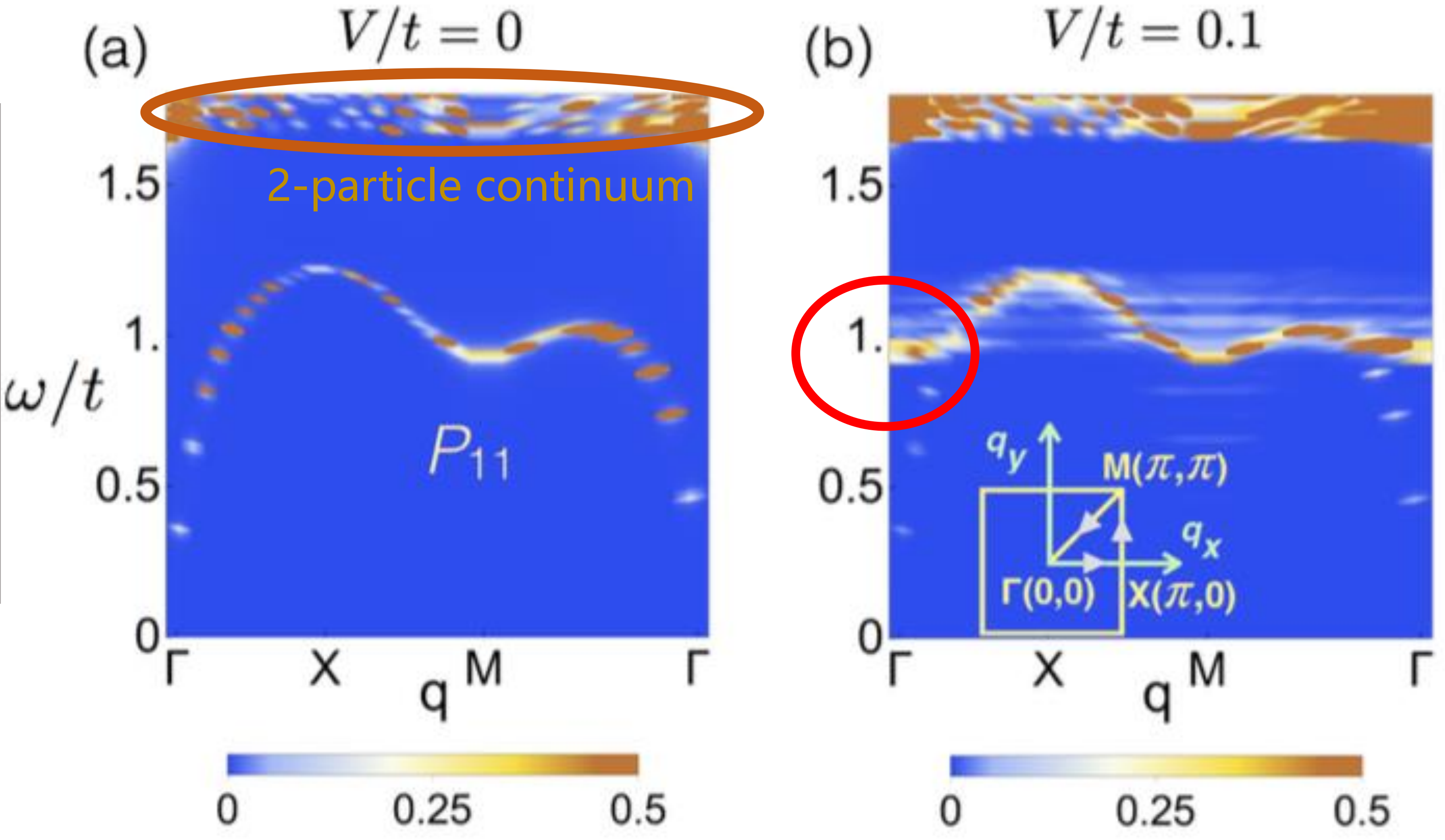
$$\boxed{S = S_0 + \sum_{ij}\sum_{\omega_n} \begin{pmatrix} \eta(i,i\omega_n) & \theta(i,i\omega_n) \end{pmatrix} \begin{pmatrix} {D^{-1}}_{11}(i,j,i\omega_n) & {D^{-1}}_{12}(i,j,i\omega_n) \\ {D^{-1}}_{21}(i,j,i\omega_n) & {D^{-1}}_{22}(i,j,i\omega_n) \end{pmatrix} \begin{pmatrix} \eta(j,-i\omega_n) \\ \theta(j,-i\omega_n) \end{pmatrix}}$$

$$\Delta(r,\tau) = (\Delta_0(r) + \eta(r,\tau))e^{i\theta(r,\tau)}$$

$$\mathcal{P}_{11}(r,r',\omega) = -\frac{1}{\pi}\mathrm{Im}\langle\eta(r,\omega+i0^+)\eta(r',-\omega+i0^+)\rangle$$
$$\mathcal{P}_{12}(r,r',\omega) = -\frac{1}{\pi}\mathrm{Im}\langle\eta(r,\omega+i0^+)\theta(r',-\omega+i0^+)\rangle$$
$$\mathcal{P}_{22}(r,r',\omega) = -\frac{1}{\pi}\mathrm{Im}\langle\theta(r,\omega+i0^+)\theta(r',-\omega+i0^+)\rangle$$

Higgs mode revealed by disorder!

In the clean case (V=0) no spectral weight at q=0 below 2-particle continuum



Weak disorder a non-dispersive mode appears at q=0 at finite energy below 2-particle continuum

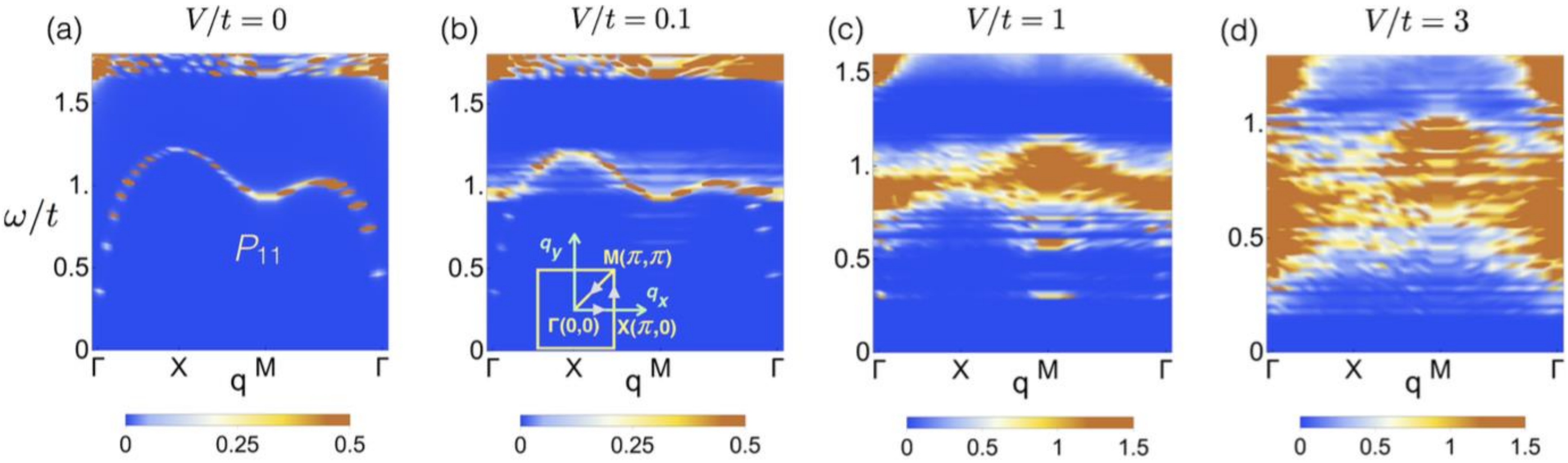
Higgs mode

$$\Delta(r,\tau) = (\Delta_0(r) + \eta(r,\tau))e^{i\theta(r,\tau)}$$

$$P = \mathcal{P}_{11} + \Delta\Delta\mathcal{P}_{22} + \Delta\mathcal{P}_{12} + \Delta\mathcal{P}_{21}$$

$$\mathcal{P}_{11}(r,r',\omega) = -\frac{1}{\pi}\mathrm{Im}\langle\eta(r,\omega+i0^+)\eta(r',-\omega+i0^+)\rangle$$

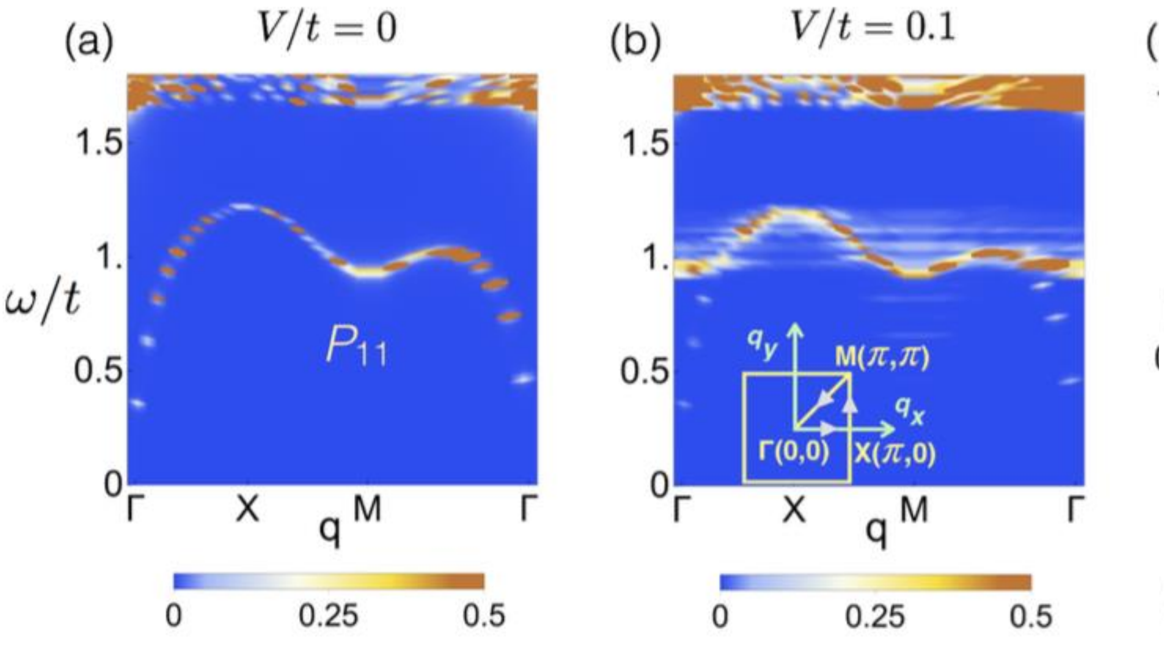
Fate of Higgs mode with increasing disorder

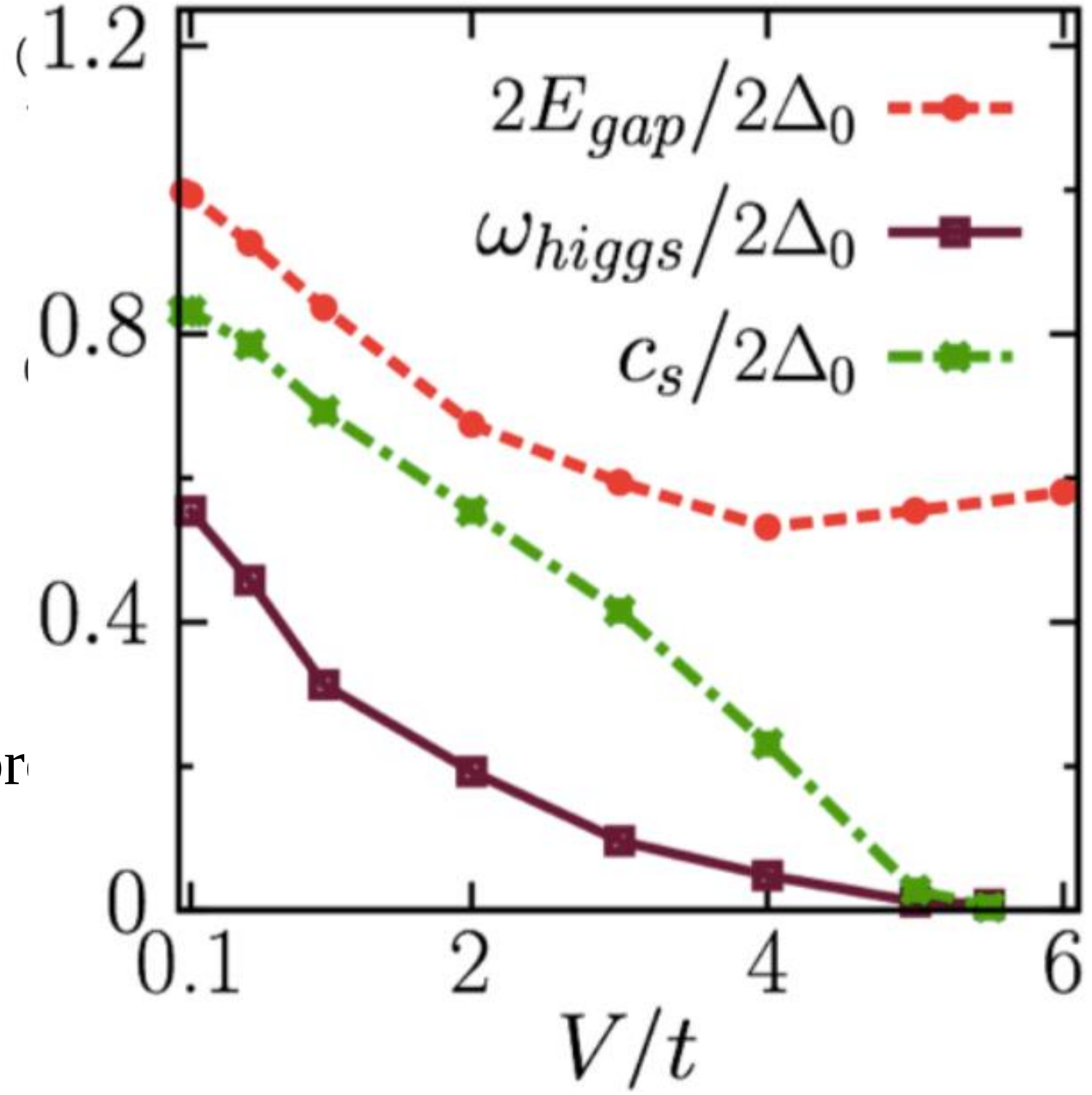


- ☑ With increasing disorder the mode broadens and the threshold for Higgs weight decreases with disorder

$$P = \mathcal{P}_{11} + \Delta\Delta\mathcal{P}_{22} + \Delta\mathcal{P}_{12} + \Delta\mathcal{P}_{21}$$

Fate of Higgs mode with increasing disorder

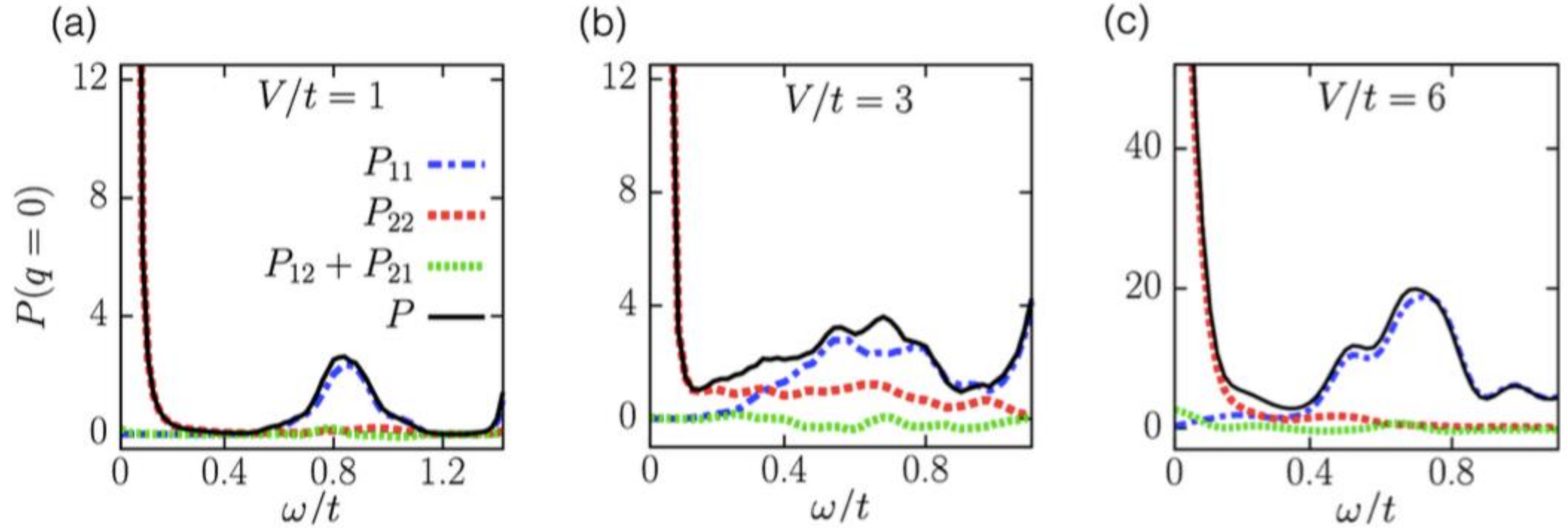




- With increasing disorder the mode br
  weight decreases with disorder

$$P = \mathcal{P}_{11} + \Delta\Delta\mathcal{P}_{22} + \Delta\mathcal{P}_{12} + \Delta\mathcal{P}_{21}$$

Two particle Correlation Function

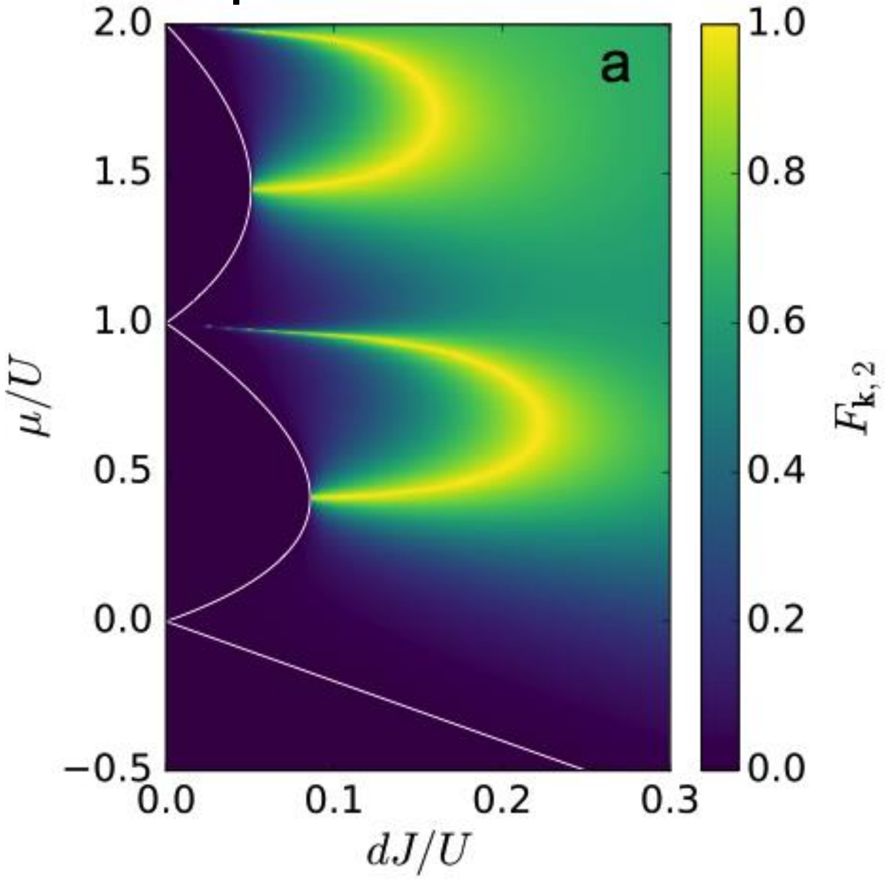


- Dominated by phase fluctuation at low energy
- Amplitude mode contributes at finite energy, spectrally well separated from phase pile-up
- Amplitude-phase mixing is small and of both signs averaging ~ to zero

M-EELS couples to q- and energy- resolved density fluctuations, e.g. Abbamonte's group Science 358 (6368), 1314-1317

Two ways to push the Higgs mode below the continuum

Bose Hubbard Model:
emergent particle-hole symmetry away from Lorentz invariant points



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